Discrete choice (1)

Applied Econometrics for Spatial Economics

Hans Koster

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tinbergen institute

1. Introduction

- Hans Koster → URBANECONOMICS.NL
  - Professor of Urban Economics and Real Estate
  - h.koster@vu.nl
  - Lectures
  - Programme director of STREEM (streem.sbe@vu.nl)

- Materials
  - All course materials, lecture slides, etc. can be accessed via www.urbaneconomics.nl/aese
  - If there is anything unclear, let me know!

1. Introduction

- This course
  - Learn about advanced tools and techniques important for spatial economics
    ➔ No theory – an applied course!

- Do not hesitate to ask questions during the class!

- Notation on slides
  - Most important concept are underlined
  - Questions (via Menti), exercises and applications
    → On red slides

1. Introduction

- Topics:
  1. Discrete choice
     - Random utility framework, estimating binary and multinomial regression models
  2. Spatial econometrics
     - Spatial data, autocorrelation, spatial regressions
  3. Identification
     - Research design, IV, OLS, RDD, quasi-experiments, standard errors
  4. Hedonic pricing
     - Theory and estimation
  5. Quantitative spatial economics
     - General equilibrium models in spatial economics

1. Introduction

Wednesday

09:30-10:30	Lecture 1	Discrete Choice I (The random utility framework)
10:45-11:45	Lecture 2	Discrete Choice II (Estimating discrete choice models)
12:00-13:00	Lecture 3	Spatial Econometrics I (Spatial data)
14:00-15:30	Tutorial 1	Assignment 1

Thursday

09:30-10:30	Lecture 4	Spatial Econometrics II (Spatial autocorrelation)
10:45-11:45	Lecture 5	Spatial Econometrics III (Spatial regressions)
12:00-12:30	Lecture 6	Identification I (Research design)
13:30-14:00	Tutorial 2	Discussion of Assignment 1
14:00-15:00	Tutorial 3	Assignment 2

Friday

09:30-10:00	Lecture 7	Identification II (RCTs, OLS, IV, quasi-experiments)
10:00-10:30	Lecture 8	Hedonic pricing I (Theory)
10:45-11:45	Lecture 9	Hedonic pricing II (Estimation)
12:00-12:30	Tutorial 4	Discussion of Assignment 2

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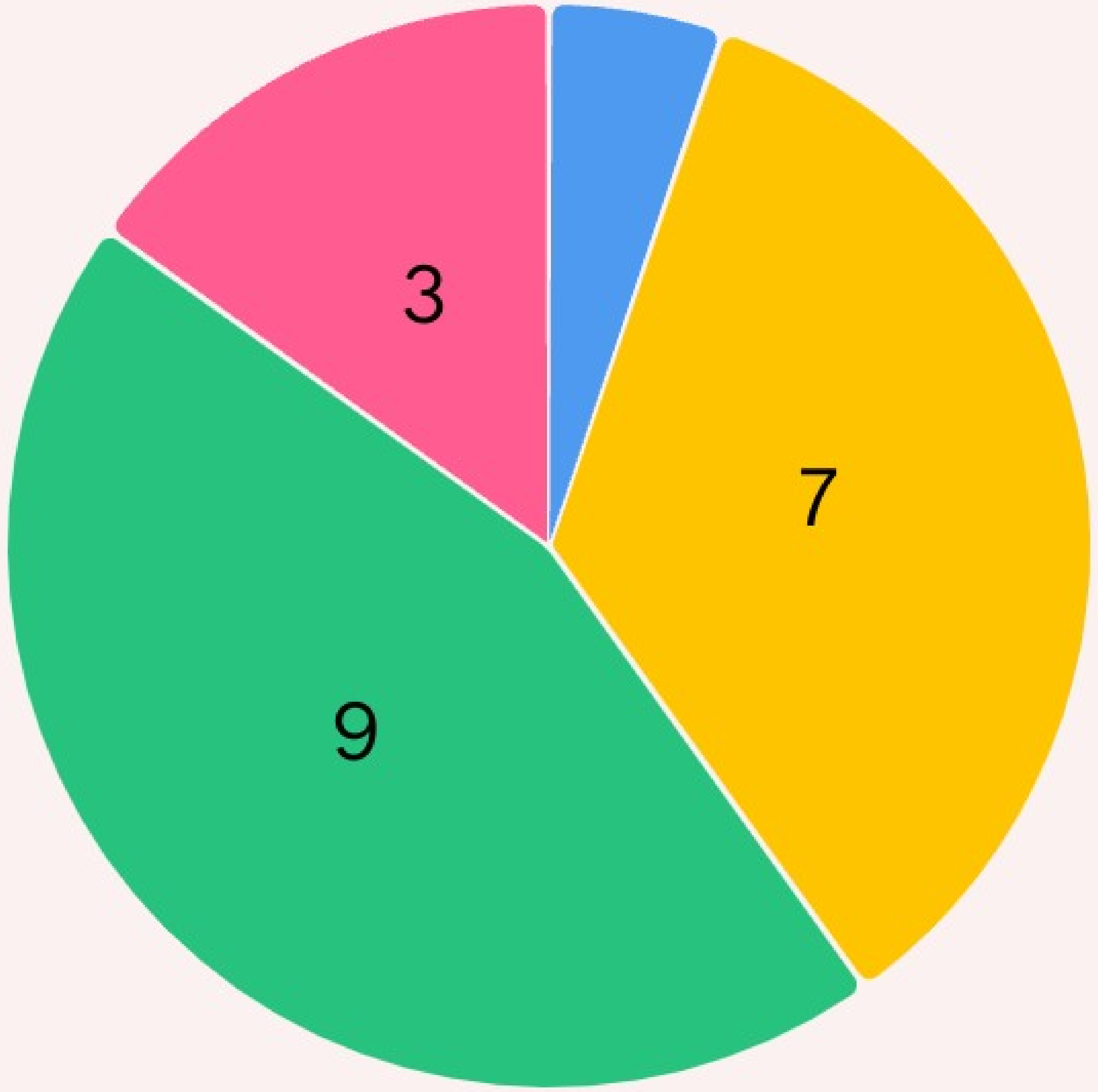
What is your highest achieved degree?



1	BSc
7	MSc
9	PhD
3	I am currently pursuing a PhD

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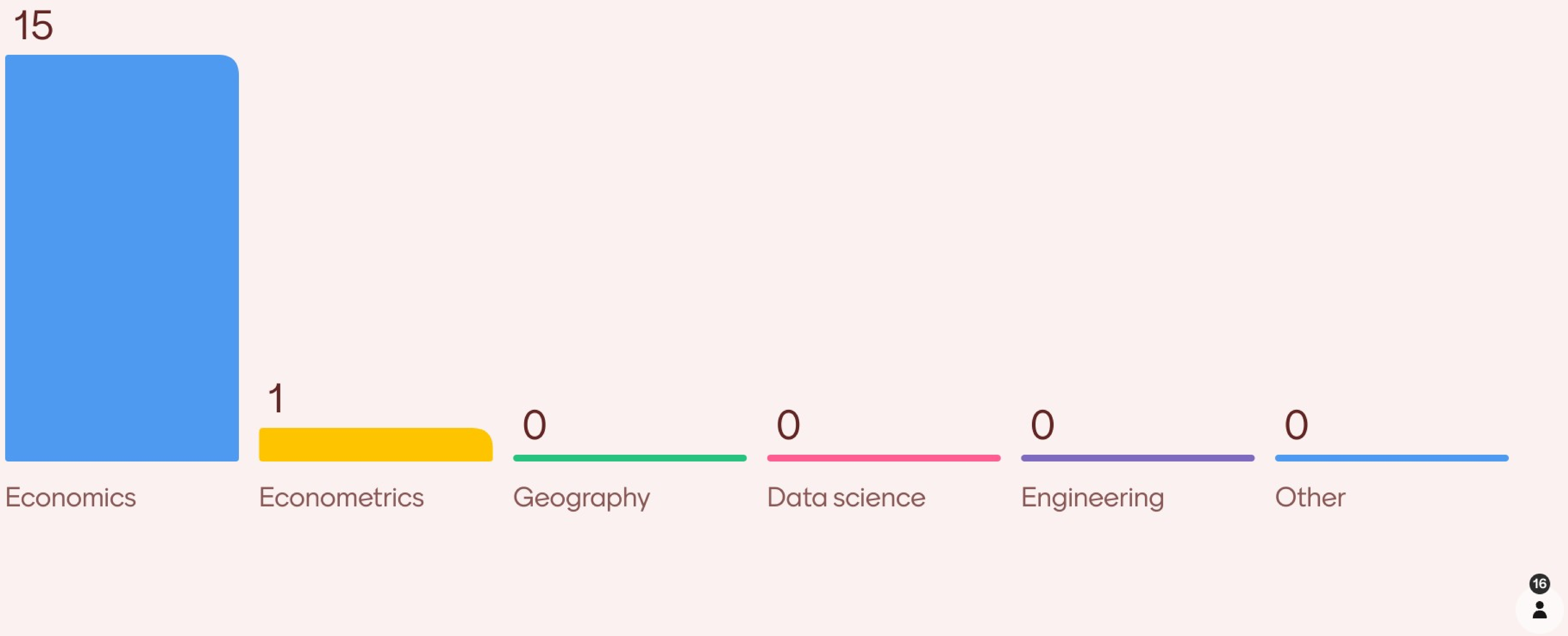
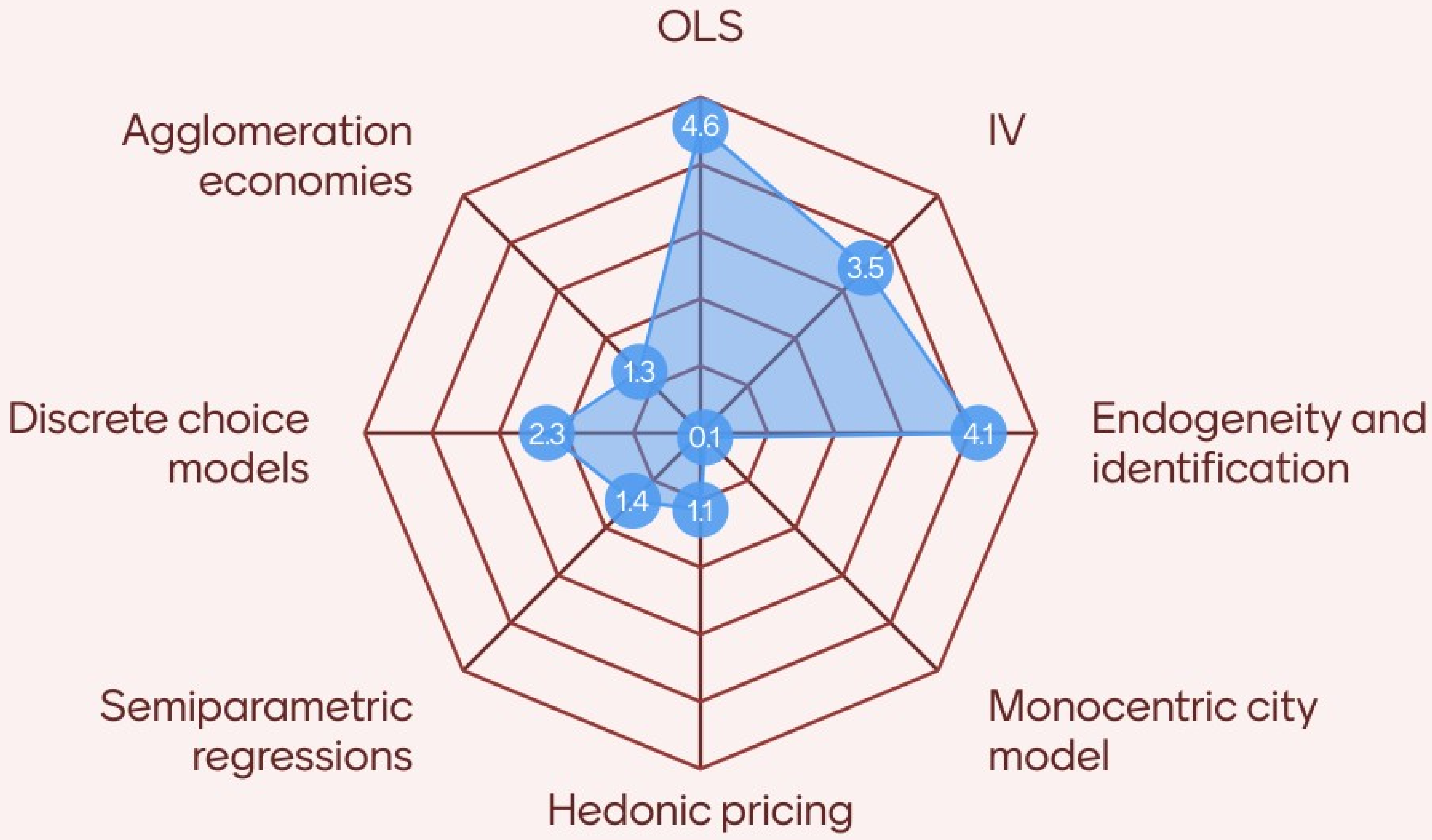
What is your main field?

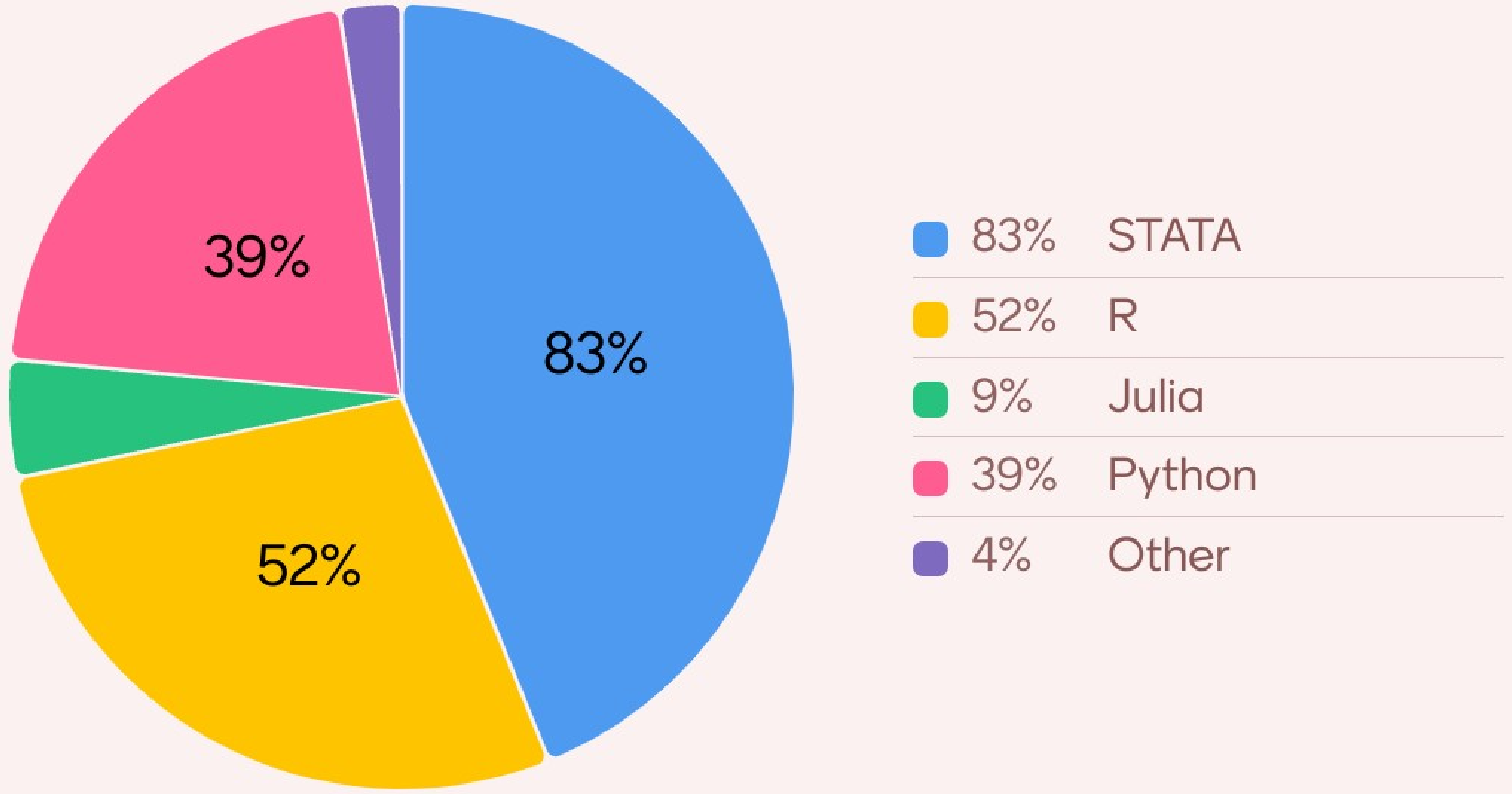


I have attended the following courses:

Please let me know how familiar you are with the following concepts:



I have experience in:



1. Introduction

1. Introduction

1. Introduction

- If you have data at your disposal you may use these data to answer practical questions:
  - What factors influence the carrier's selection of a port?
  - Which mode do people prefer to travel from A to B?
  - Where do people want to live?

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1. Introduction

- *Continuous* choice: estimate marginal changes in behaviour
  - E.g. "when fuel price increases by 10%, the demand for fuel will decrease by 2%"
  - Standard micro-economic theory applies

- Transport demand often has a discrete (binary) nature
  - Some x impacts a discrete y
  - Then use discrete choice methods

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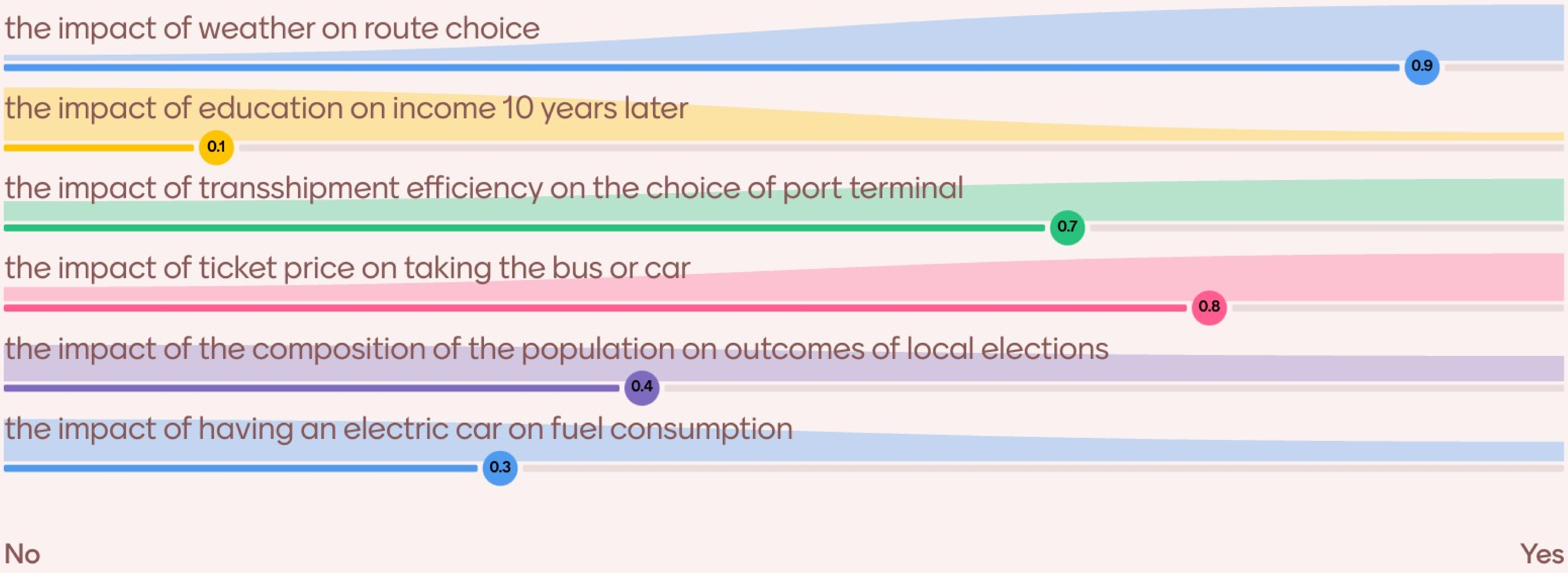
Would you use discrete choice methods when you aim to test:



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1. Introduction

- **Discrete choice methods**
  - ***Dependent* variable y_i is discrete**

- **Why not use OLS?**

- **Let's have the standard OLS equation**

  $$y_i = \beta x_i + \epsilon_i \qquad (1)$$

  where i indexes the individual

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1. Introduction

- OLS may be consistent for binary choice
  - *But,* y_i *(and therefore* ϵ) is *not* normally distributed
- Horrace and Oaxaca (2006)
  - Leads to biased and inconsistent estimates if $\hat{y}_i$ lies 'often' outside the [0,1] interval
  - I show later today why that is an issue…
- OLS does not necessarily provides a link with economic theory
- Not suitable for multinomial choice

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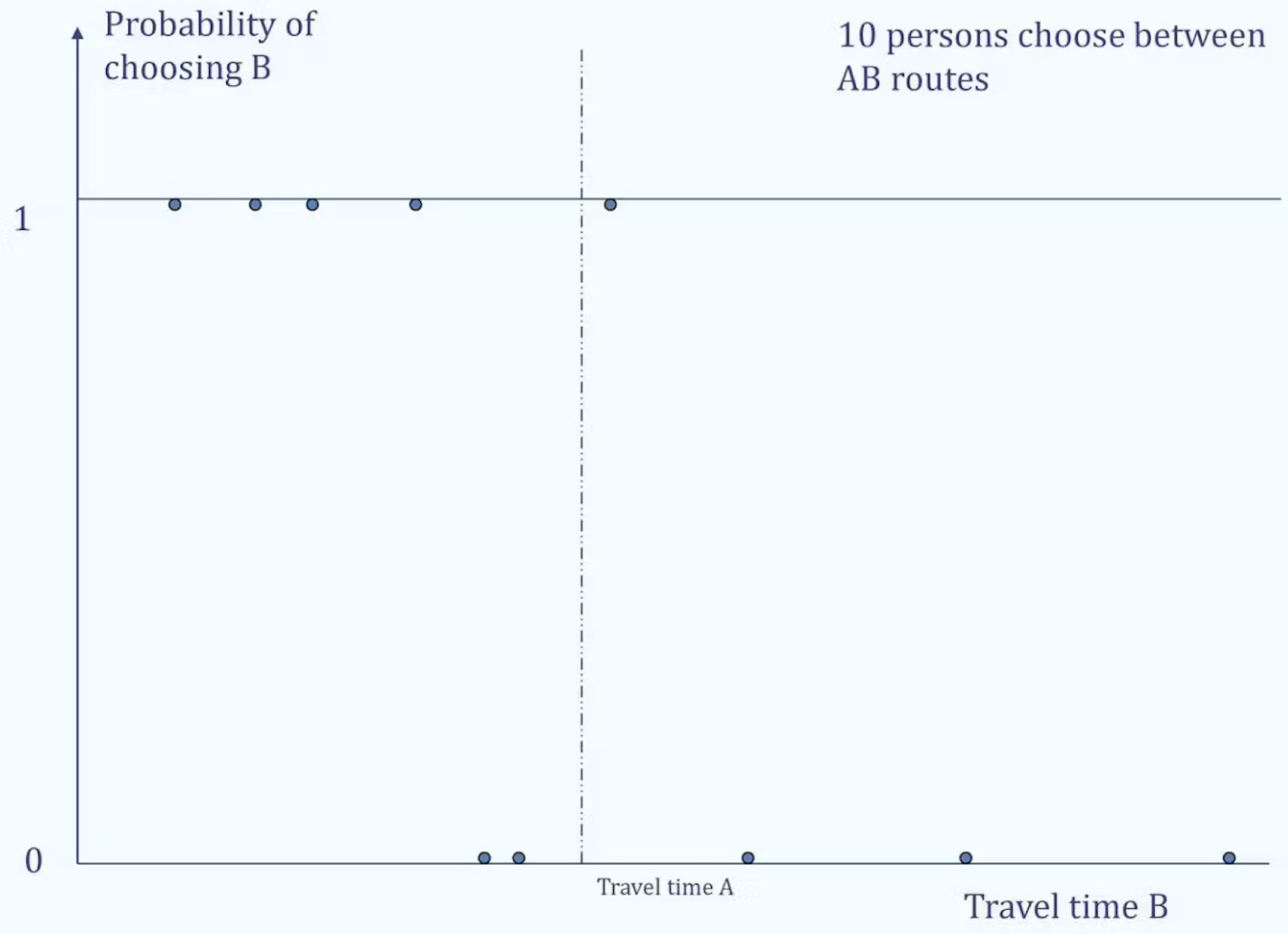
2. The random utility framework

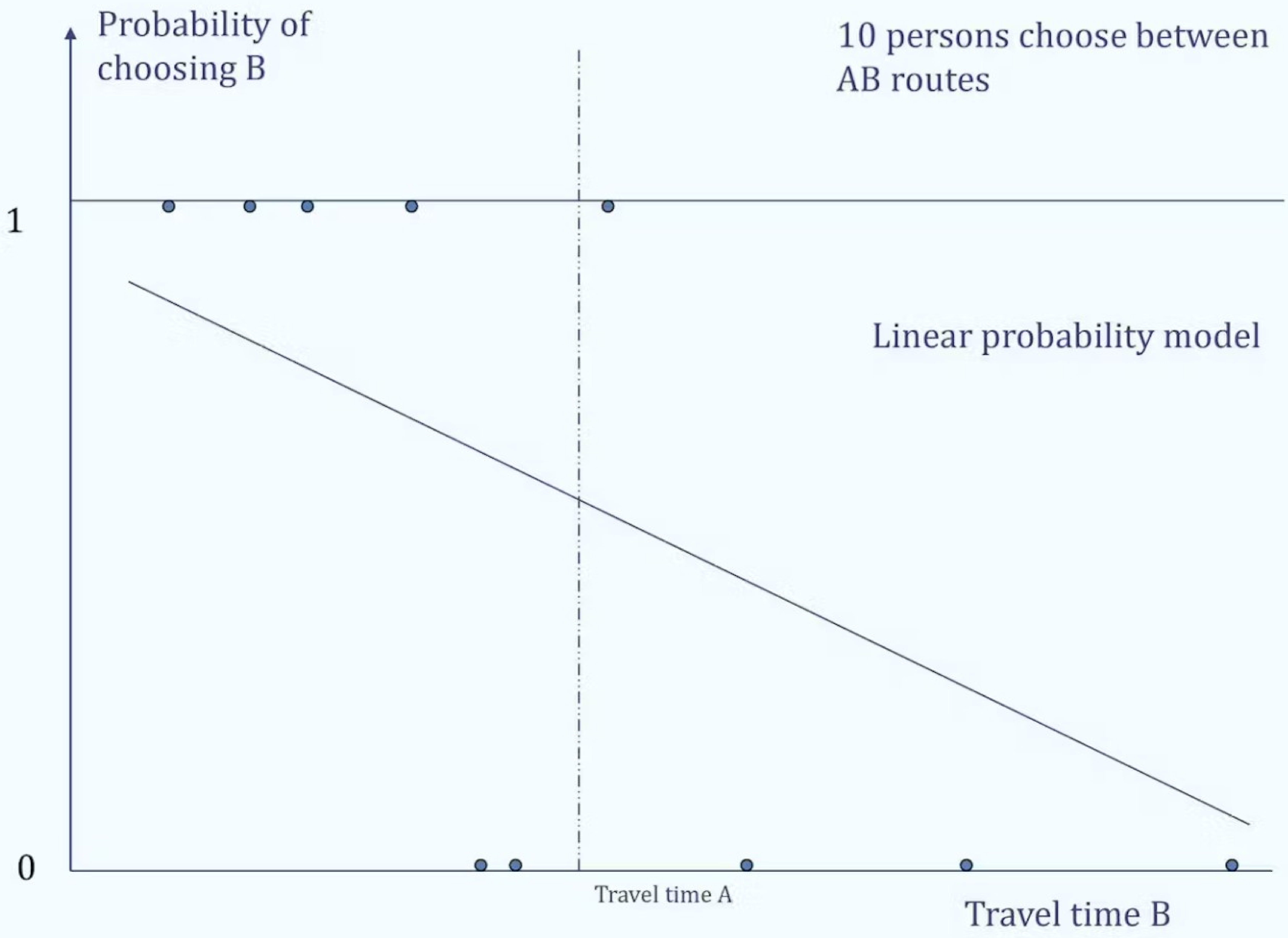
- Consider you have 10 individuals that choose between two routes A en B
- Travel time of A is 9 and of B is 10 minutes
- Some people take route B
  - E.g. because they like particular features of a route, or they misjudge the travel time

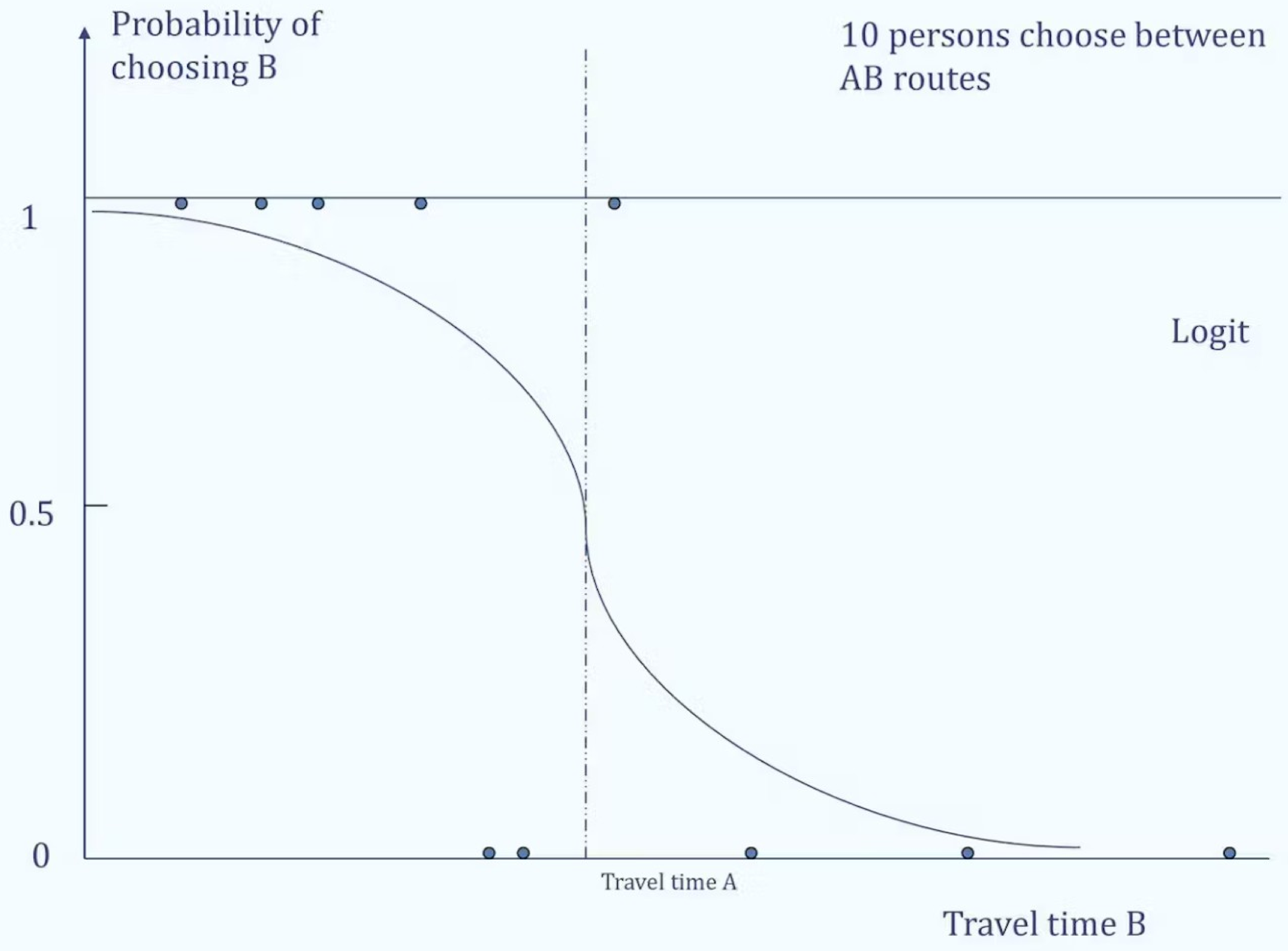
- Let's do a regression of whether or not you have chosen B on the difference between the travel time of A and B:

  $y_B = f\big(\alpha + \beta(travel\ time_B - travel\ time_A)\big) + \epsilon$

  where $y_B = 1$ if you choose B and zero otherwise


Probability of
choosing B
10 persons choose between
AB routes
1
0
Travel time A
Travel time B


Probability of
choosing B
10 persons choose between
AB routes
1
Linear probability model
0
Travel time A
Travel time B



Probability of choosing B
10 persons choose between AB routes
1
0.5
0
Logit
Travel time A
Travel time B

2. The random utility framework

- **Indirect utility may be given by:**

  $U_{iA} = V_A(travel\ time_A) + \epsilon_{iA}$ (2)

  $U_{iB} = V_B(travel\ time_B) + \epsilon_{iB}$ (3)

  - V_A, V_B ➔ **deterministic utility**

- **Random terms:** $\epsilon_{iA}, \epsilon_{iB}$**:** **random taste variation**
  - **Random utility model (RUM)**
  - **Note that the levels of** U_{iA} **and** U_{iB} **are not directly observed!**

- $\Pr(Y = A) = \Pr(U_{iA} > U_{iB})$
- $\Pr(V_A + \epsilon_{iA} > V_B + \epsilon_{iB}) = \Pr(V_A - V_B > \epsilon_{iB} - \epsilon_{iA})$

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2. The random utility framework

- Two things are unknown
  - Which distribution for ϵ's?
  - What is the functional form for V_A and V_B?

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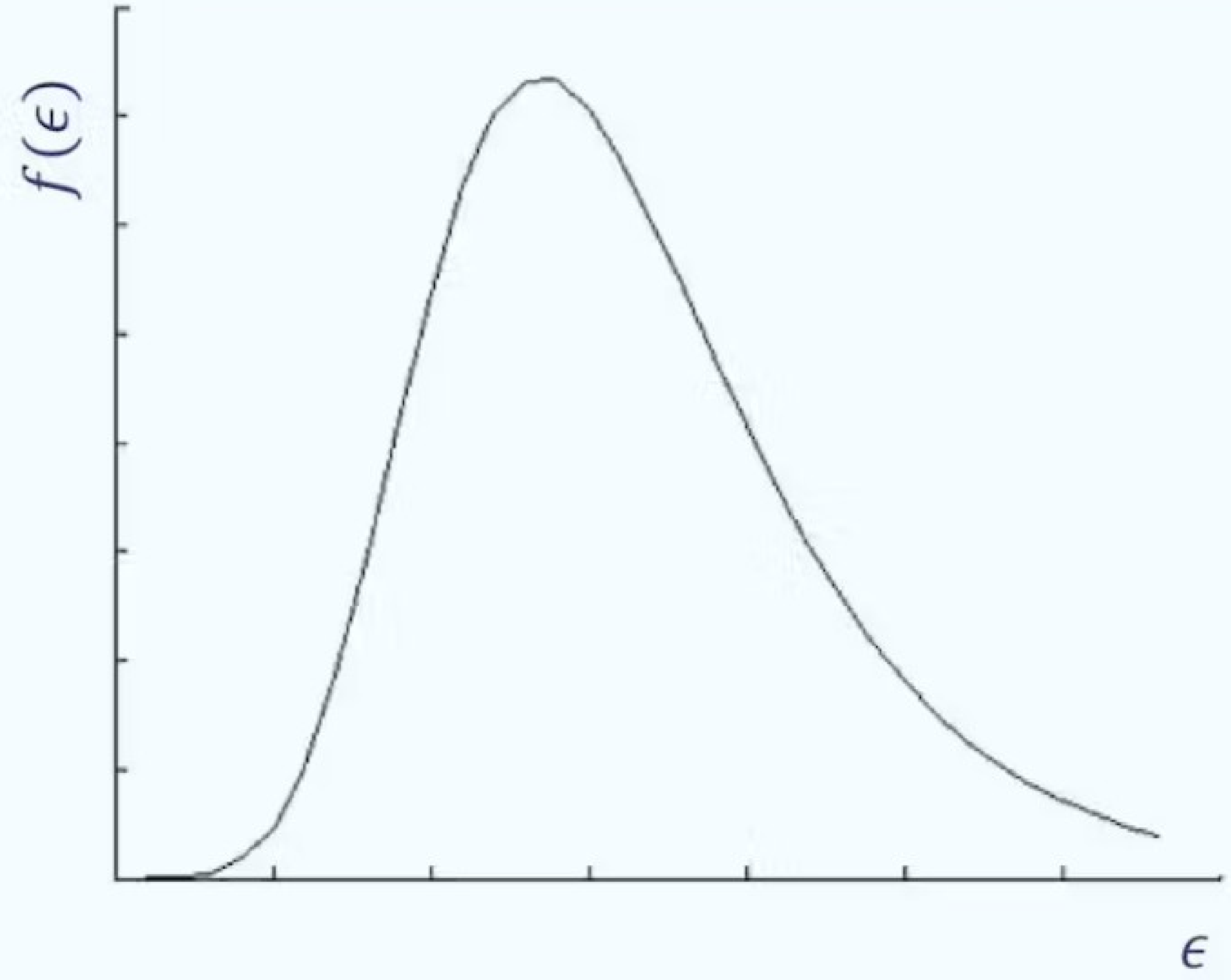
2. The random utility framework

- **Which distribution for ϵ's?**
  - **ϵ's are unobserved**
  - **You draw them from a distribution**

  - **Logit: Extreme Value Type I distribution**



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- **Which distribution for ϵ's?**
  - **Extreme Value Type I distribution**
  - **Generates simple closed-form solutions!**

    $\rightarrow \quad \Pr(V_A - V_B > \epsilon_{iB} - \epsilon_{iA})$
  - **Daniel McFadden (1964)**

2. Random utility framework

- **It appears that:**

$$\Pr(Y = A) = \frac{e^{V_A}}{e^{V_A} + e^{V_B}} \qquad (4)$$

- **With two alternatives this can be written as:**

$$\Pr(Y = A) = \frac{1}{1 + e^{V_B - V_A}}$$

2. The random utility framework

- **It appears that:**

$$\Pr(Y = A) = \frac{e^{V_A}}{e^{V_A} + e^{V_B}} \tag{4}$$

→ **Show that (4) can be written as:**

$$\Pr(Y = A) = \frac{1}{1 + e^{V_B - V_A}}$$

→ **Compute probability when $V_A = V_B$**

→ **What happens when $V_A \gg V_B$**

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(1) Show that $Pr(Y = A) = \frac{e^{V_A}}{e^{V_A} + e^{V_B}} = \frac{1}{1+e^{V_B - V_A}}$. (2) Compute probability when $V_A = V_B$ and when $V_A \gg V_B$.

10

I'm ready!

10

2. The random utility framework

- **It appears that:**

$$\Pr(Y = A) = \frac{e^{V_A}}{e^{V_A} + e^{V_B}} \qquad (4)$$

→ **Show that (4) can be written as:**

$$\Pr(Y = A) = \frac{1}{1 + e^{V_B - V_A}}$$

Divide everything by $\exp(V_A)$*, and note that* $\exp(V_B - V_A) = \exp(V_B)/\exp(V_A)$

→ **Compute probability when $V_A = V_B$**

Probability = 0.5

→ **What happens when $V_A \gg V_B$**

Probability = 1

2. The random utility framework

- **Which functional form for V_A and V_B?**
  - **Can be any function**
  - **Linear function is often assumed**
  - **Can be extended with multiple variables**

    $$U_{jA} = \beta p_{jA} + \kappa t_{jA} + \epsilon_{jA} \quad (5)$$

    $$U_{jB} = \beta p_{jB} + \kappa t_{jB} + \epsilon_{jB} \quad (6)$$

    where p_{jA} is the price of a trip and t_{jA} is travel time of alternative j
  - $\beta < 0, \kappa < 0$

- **Recall (from previous slide):**
  - $\Pr(Y = A) = \frac{1}{1+e^{\beta(p_{jB}-p_{jA})+\kappa(t_{jB}-t_{jA})}}$

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3. Value of time

- **Important concept in Transport Economics: *Value of Time* (VOT)**
  - **"How much are you willing to pay to reduce your travel time with one hour, *holding utility constant*"**

- **Let's take the deterministic utility function**

$$U_{jA} = \beta p_{jA} + \kappa t_{jA} + \varepsilon_{jA} \qquad (7)$$

- **When t_{jA} is measured in hours, the VOT can be written as κ / β**

3. Value of time

- **Important concept in Transport Economics:** ***Value of Time*** **(VOT)**
  - **"How much are you willing to pay to reduce your travel time with one hour,** ***holding utility constant*****"**

- **Let's take the deterministic utility function**
  $U_{iA} = \beta p_{iA} + \kappa t_{iA} + \gamma x_{iA} + \varepsilon_{iA}$
  - $\Delta U_{iA} = 0$ (7)

→ **Show that when t_{iA} is measured in hours, the VOT can be written as κ/β**

With $U_{iA} = \beta p_{iA} + \kappa t_{iA} + \gamma x_{iA} + \varepsilon_{iA}$, show that when t_{iA} is measured in hours, the Value of Time can be written as κ/β.

0

I'm ready!

3. Value of time

→ **Show that when t_{iA} is measured in hours, the VOT can be written as κ/β**

Let's look at a change in utility with respect to time and price:

$$\frac{\partial U_{iA}}{\partial t_{iA}} = \kappa$$

$$\frac{\partial U_{iA}}{\partial p_{iA}} = \beta$$

- **We hold utility constant, so $\Delta U_{iA} = 0$**
- $-\beta \Delta p_{iA} = \kappa \Delta t_{iA}$
- $\Delta t_{iA} = -1$ **(WTP for one hour reduction in travel time)**
- $\Delta p_{iA} = \frac{\kappa}{\beta}$

3. Value of time

- Value of time is often used in cost benefit analyses

- VOT depends on trip purpose
  - Business €26.25/h
  - Commuting €9.25/h
  - Social purpose €7.50/h

- VOT depends on income
  - About 50% of net income

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3. Value of time

- Other applications
  - Value of a Statistical Life (VSL)
    The VSL is the local tradeoff between fatality risk and money
  - Value of schedule delay (VSD)
    The VSD is the local tradeoff between being too early/late and money
  - Etc.
  - … What is necessary is a cost/reward parameter in the discrete choice experiment

4. Multiple alternatives

- **The choice probability for two alternatives:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B}}$$

- **Usually there are more alternatives in the choice set**
  - **Train, bus, car**
  - **Rotterdam, Antwerp, Hamburg**
  - **Routes to the VU**

- **Simply extend the logit formula:**

$$\Pr(Y = A) = \frac{e^{\beta x_A}}{e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}}$$

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4. Multiple alternatives

- **The aggregate utility derived from the choice set is summarised by the logsum:**

  $$E[CS] = \frac{1}{\upsilon}\ln\left(e^{\beta x_A} + e^{\beta x_B} + e^{\beta x_C}\right)$$

  - **υ is the marginal utility of income**
  - **Can be used in welfare estimates**

- **Assume $\beta x_A = \beta x_B = 10$**

- **Now alternative C is added and $\beta x_C = 1$**

- **The average utility per alternative decreases from 10 to 7 but E[CS] increase**
  - **'Love of variety' effect**

4. Multiple alternatives

- **Property of logit formula:**
  - **The *ratios* of choice probabilities for A and B do not depend on whether or not C is in the choice set**
  - **Independence of irrelevant alternatives**
  - $$\frac{\Pr(Y=A)}{\Pr(Y=B)} = \frac{\left(\frac{e^{\beta x_A}}{e^{\beta x_A}+e^{\beta x_B}+e^{\beta x_C}}\right)}{\left(\frac{e^{\beta x_B}}{e^{\beta x_A}+e^{\beta x_B}+e^{\beta x_C}}\right)} = \frac{e^{\beta x_A}}{e^{\beta x_B}}$$
  - **Let's find out whether this is a desirable property…**

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4. Multiple alternatives

- ‘The *Red Bus-Blue Bus*’ problem

- Choice set 1: Train, red bus, blue bus
- Assume market shares are 70, 15 and 15%

	Train	Red bus	Blue bus
V	2.54	1	1
Prob	0.700	0.150	0.150

- Choice set 2: Train, red bus, so:

	Train	Red bus
V	2.54	1
Prob	0.823	0.177

- Probability to take the bus in choice set 2 is

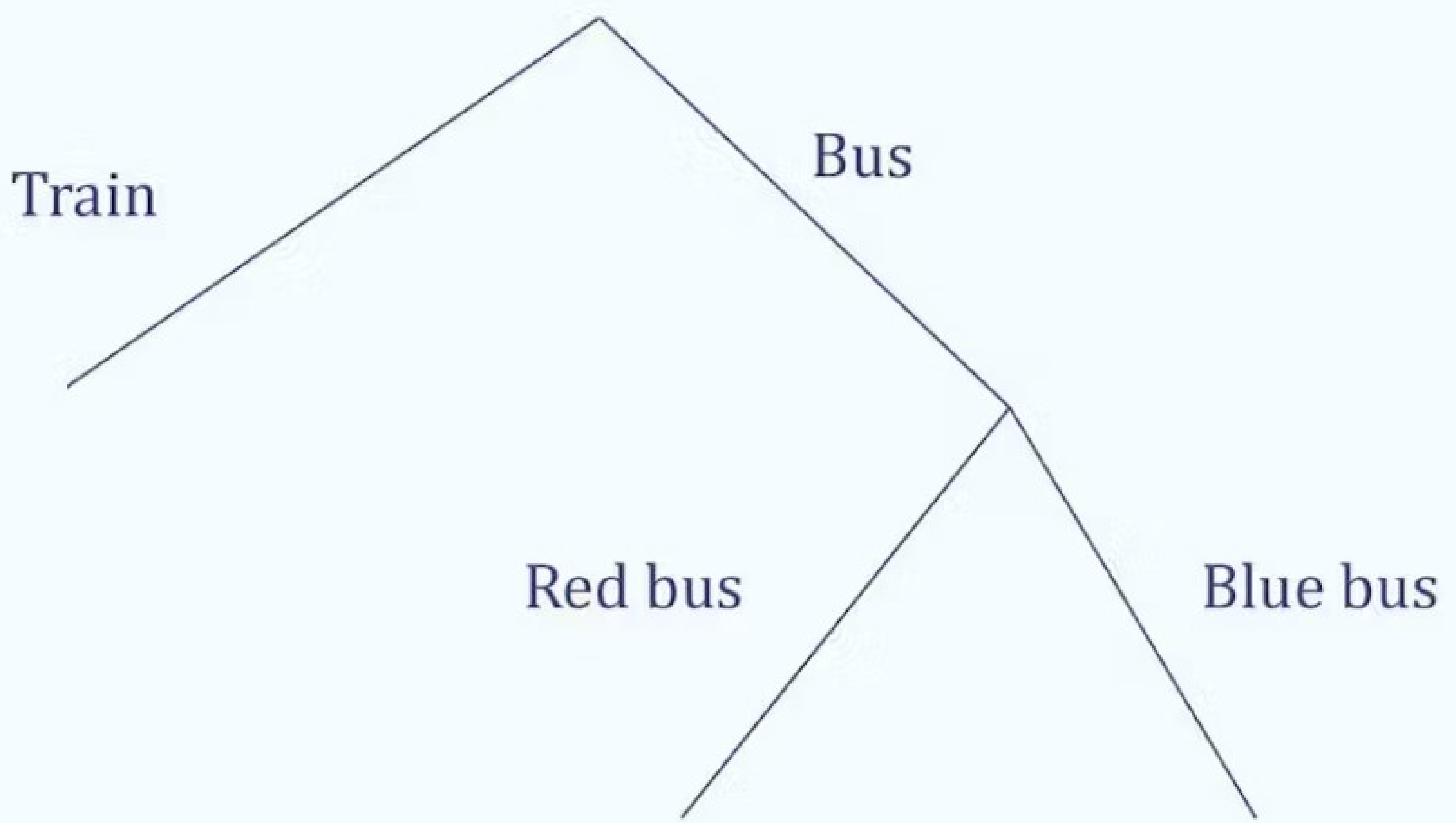
$$\frac{e^1}{e^{2.54}+e^1} = 0.177$$

  - Higher probability – not very realistic as red buses and blue buses are identical

4. Multiple alternatives

- **So, when some alternatives are more similar than other alternatives, the use of multinomial choice model may be misleading**

- **Use nested logit!**



4. Multiple alternatives

- Nested logit takes into account correlation between alternatives
  - But define nests yourself!

4. Multiple alternatives

- Nested logit takes into account correlation between alternatives
  - But define nests yourself!

→ Assume you have four travel modes: car, carpool, train and bus. Please define a meaningful nested structure

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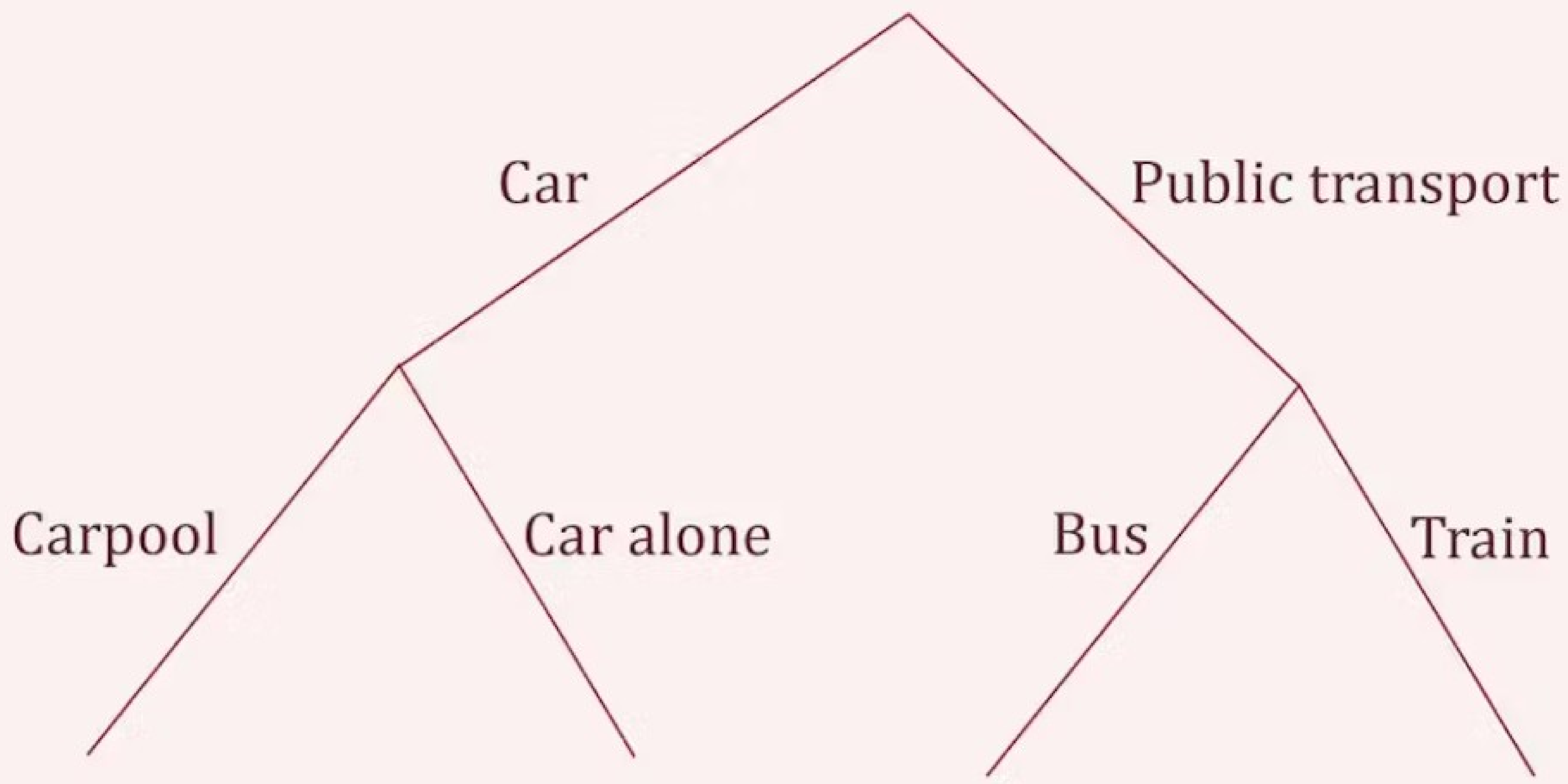
Assume you have four travel modes: car, carpool, train and bus. Please define a meaningful nested structure.

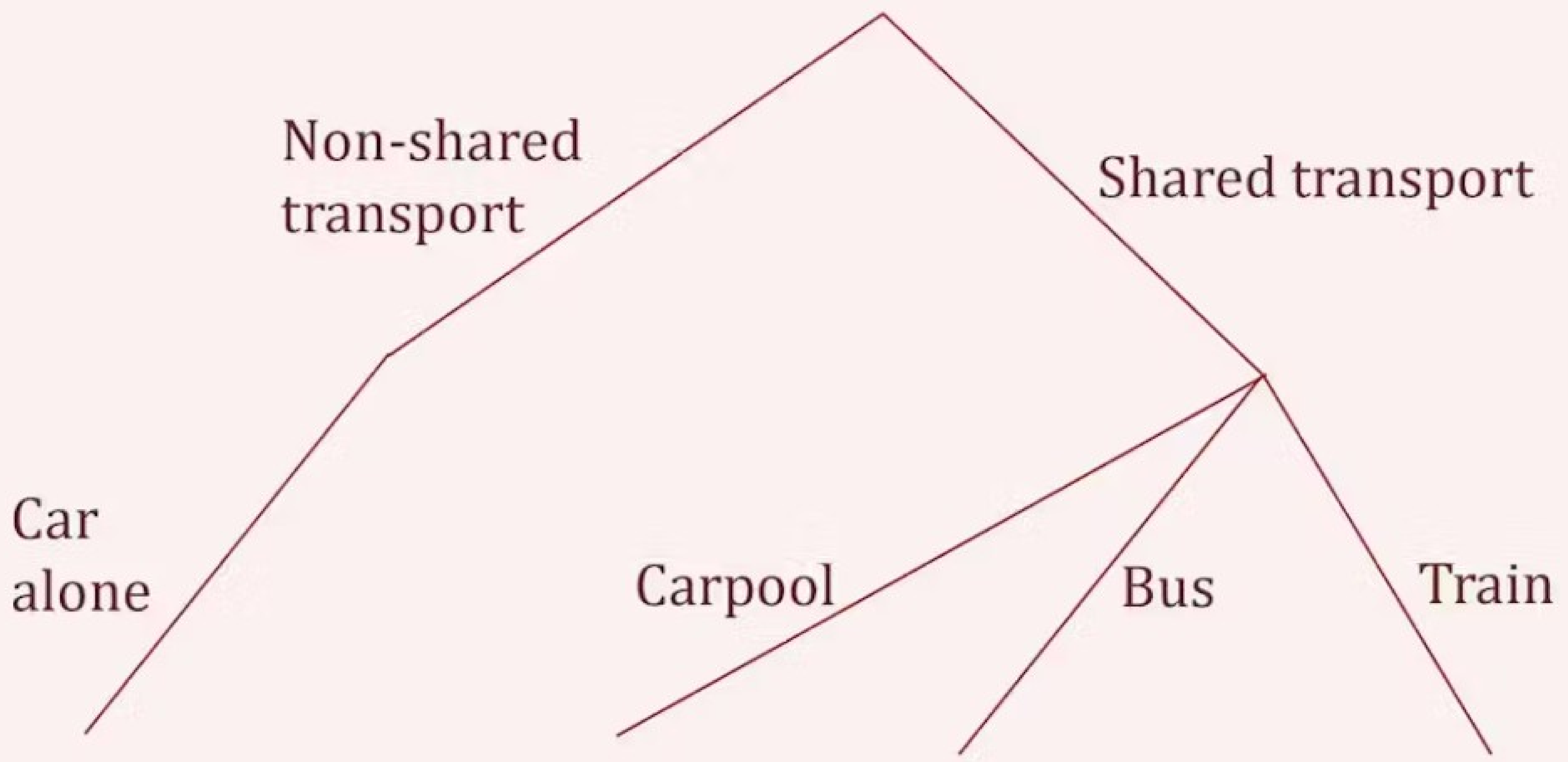
4. Multiple alternatives

- **Example nested structure:**



4. Multiple alternatives

- **Example nested structure:**



2. The random utility framework

- **Let us define utility as follows:**

  $U_{jg} = V_j + W_g + \epsilon_{jg}$

  V_j **only differs within nests between alternatives** j

  W_g **only differs between nests** g

2. The random utility framework

- **We may write the probability to choose an alternative:**
  - $\Pr(d_j = 1) = \Pr(g) \cdot \Pr(j \mid g)$
  - $\Pr(j \mid g) = \frac{e^{V_j/\lambda_g}}{\sum_{k \in g} e^{V_k/\lambda_g}}$
  - $\Pr(g) = \frac{e^{W_g + \lambda_g I_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}} + \lambda_{\tilde{g}} I_{\tilde{g}}}}$

    with $I_g = \log\left(\sum_{j \in g} e^{V_j/\lambda_g}\right)$

- $\lambda_g = 1 \Rightarrow$ **no correlation (multinomial logit)**
- $\lambda_g \to 0 \Rightarrow$ **perfect correlation (red bus/blue bus)**

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2. The random utility framework

- **When j and k are in the same nest:**

$$\frac{\Pr(d_j = 1)}{\Pr(d_k = 1)} = \frac{e^{W_g + V_j / \lambda_g}}{e^{W_g + V_k / \lambda_g}} = \frac{e^{W_g + V_j}}{e^{W_g + V_k}} = \frac{e^{V_j}}{e^{V_k}}$$

  - **IIA property holds *within* nests**

- **When $\lambda_g \to 0$:**
  - $\Pr(j \mid g) = \frac{e^{V_j / \lambda_g}}{\sum_{k \in g} e^{V_k / \lambda_g}} = 1/k_g$
  - $\Pr(g) = \frac{e^{W_g + \lambda_g I_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}} + \lambda_{\tilde{g}} I_{\tilde{g}}}} = \frac{e^{W_g}}{\sum_{\tilde{g}} e^{W_{\tilde{g}}}}$
  - **Hence, multinomial logit *between* nests**

- So, nested logit probability depends on
  - Probability to choose a nest
  - Probability to choose an alternative within the nest

- Note that Nested Logit does not imply a *sequential* choice

5. Summary

Today:

- **How to deal with a binary dependent variable?**
- **Links to economic theory with random taste variation**
  - **Random utility model**
  - **Assume distribution of ϵ_i**
  - **Extreme Value Type I is convenient**
- **Nested logit relaxes the *Independence of Irrelevant Alternatives* assumption**

Discrete choice (1)

Applied Econometrics for Spatial Economics

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