Spatial econometrics (2)

Applied Econometrics for Spatial Economics

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tinbergen institute

1. Introduction
2. Spatial autocorrelation
3. Spatial regressions
4. Summary

1. Introduction

- Topics:
  1. Discrete choice
     - Random utility framework, estimating binary and multinomial regression models
  2. Spatial econometrics
     - Spatial data, autocorrelation, spatial regressions
  3. Identification
     - Research design, IV, OLS, RDD, quasi-experiments, standard errors
  4. Hedonic pricing
     - Theory and estimation
  5. Quantitative spatial economics
     - General equilibrium models in spatial economics

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Wednesday

09:30-10:30	Lecture 1	Discrete Choice I (The random utility framework)
10:45-11:45	Lecture 2	Discrete Choice II (Estimating discrete choice models)
12:00-13:00	Lecture 3	Spatial Econometrics I (Spatial data)
14:00-15:30	Tutorial 1	Assignment 1

Thursday

09:30-10:30	Lecture 4	Spatial Econometrics II (Spatial autocorrelation)
10:45-11:45	Lecture 5	Spatial Econometrics III (Spatial regressions)
12:00-12:30	Lecture 6	Identification I (Research design)
13:30-14:00	Tutorial 2	Discussion of Assignment 1
14:00-15:00	Tutorial 3	Assignment 2

Friday

09:30-10:00	Lecture 7	Identification II (RCTs, OLS, IV, quasi-experiments)
10:00-10:30	Lecture 8	Hedonic pricing I (Theory)
10:45-11:45	Lecture 9	Hedonic pricing II (Estimation)
12:00-12:30	Tutorial 4	Discussion of Assignment 2

2. Spatial autocorrelation

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- Spatial autocorrelation between values
  - Implies $\text{cov}(x_i, x_j) = \text{E}[x_i x_j] - \text{E}[x_i] \cdot \text{E}[x_j] \neq 0$
  - Again, j refers to other locations

- Spatial autocorrelation, dependence, clustering
  - Fuzzy definitions in literature

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2. Spatial autocorrelation

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- How to measure spatial autocorrelation
  - Moran's *I*
  - Focus on one variable x (e.g. crime)

- H_0: independence, spatial randomness
- H_A: dependence
  - On the basis of adjacency, distance, hierarchy

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- **Moran's I is given by:**

$$I = \frac{R}{S_0} \times \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}} \qquad (4)$$

where R is the number of spatial units
S_0 is the sum of all elements of the spatial weight matrix
W is the spatial weight matrix
$\tilde{x} = x - \bar{x}$ is a vector with the variable of interest

- **Use row-standardised spatial weight matrix W!**
  - So that $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}}$

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- **Moran's *I***

- **Sidenote:**
  - **Please realise that $W\tilde{x}$ is a vector**
    - $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}'\tilde{x}}$

  - $\mathbf{W} \times \tilde{x} = \mathbf{W}\tilde{x}$

$$\begin{bmatrix} 0 & w_{12} & w_{13} \\ w_{21} & 0 & w_{23} \\ w_{31} & w_{32} & 0 \end{bmatrix} \times \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \end{bmatrix}$$

  - **Notation:** $\frac{x'y}{x'x} = x^T y \left(x^T x\right)^{-1}$

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- **Moran's *I***

- **Recall that** $I_S = \frac{\tilde{x}' W \tilde{x}}{\tilde{x}' \tilde{x}}$ **(standardised** I**)**
  - **Note similarity with OLS:** $\hat{\beta} = \frac{x'y}{x'x}$
  - **Hence:** $W\tilde{x} = \alpha + I\tilde{x} + \epsilon$, **where** $\alpha = 0$

- **Moran's *I* is correlation coefficient (more or less)**
  - $\approx [-1,1]$
  - **But: expectation** $\mathrm{E}[I] = -\frac{1}{N-1}$

- **Visualisation**
  - **Moran scatterplot**

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- **Moran's *I***

- **How to investigate the statistical significance of (4)?**
  - $\frac{I - \mathrm{E}[I]}{\sqrt{\mathrm{var}[I]}}$ **(5)**
  - **However, $\sqrt{\mathrm{var}[I]}$ is difficult to derive**
  - $\mathrm{E}[I] = -1/(n-1)$
  - **Assume normal distribution of I to approximate $\sqrt{\mathrm{var}[I]}$ under H_0**
  - **Or: bootstrapping/simulation**

  - **See Cliff and Ord (1973) for more details**

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- **Moran's *I***

- **Also possible: correlation to other variables:**

$$I_S = \frac{\tilde{x}' W \tilde{z}}{\tilde{x}' \tilde{x}}$$

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- **How to calculate Moran's *I* using software**
  - SPAUTOC **in STATA**
  - SPLAGVAR **in STATA**
  - SPATIAL STATISTICS TOOLBOX **in ArcGIS**

- **Alternative: Getis and Ord's *G***
  - **Most of the time only Moran's *I* is reported**

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- Let's try to answer the queston:
  "Is social deprivation spatially clustered?"

- How to determine the most deprived neighbourhoods?

- Dutch government calculated deprivation *z*-score for each neighbourhood
  - Based on housing quality, safety, perception and satisfaction
  - *Important:* the 83 most deprived neighbourhoods were selected for an investment of >€1 billion

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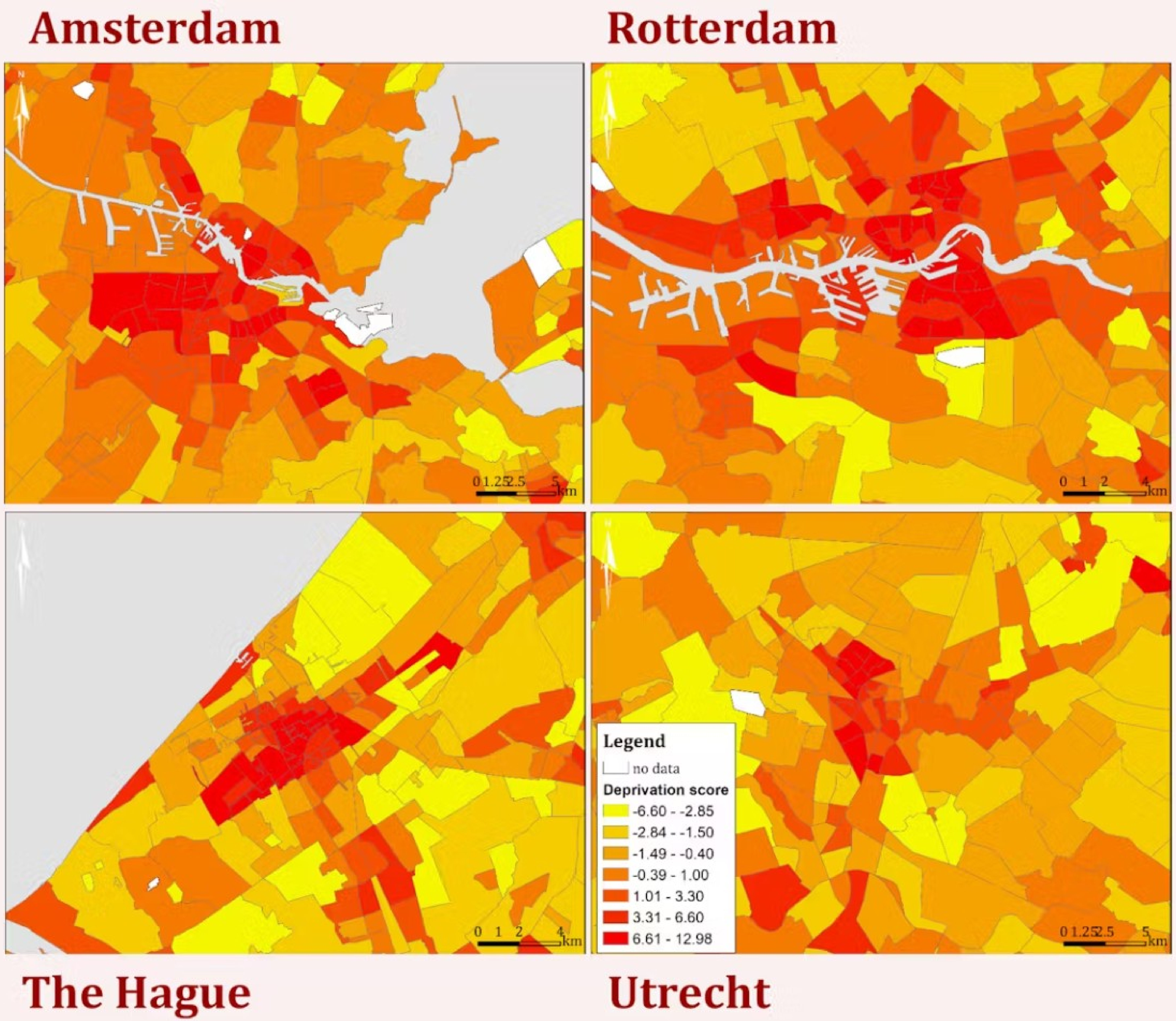
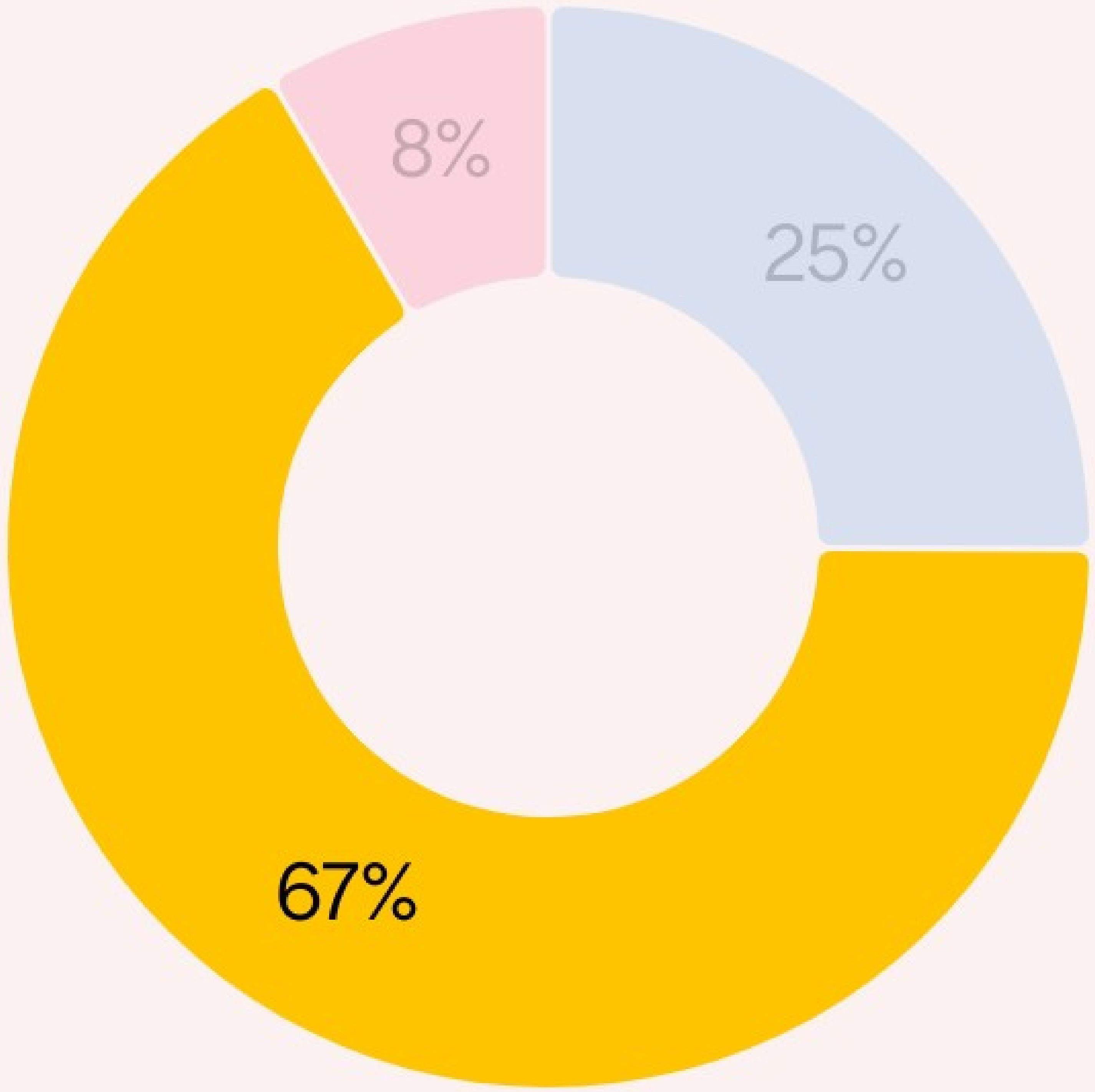


What is your hypothesis when looking at the figure



25%	Moran's I ≈ 1
67%	Moran's I ≈ 0.5
0%	Moran's I ≈ 0
8%	Moran's I ≈ -0.5
0%	Moran's I ≈ -1

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- **Determine spatial autocorrelation**
  1. **Determine distance between all neighbourhoods using centroids**
  2. **Use inverse distance function** $w_{ij} = 1/\left(d_{ij}^{\gamma}\right)$ **to determine spatial weights in weight matrix**
  3. **Calculate Moran's I:** $\mathrm{W}\tilde{\mathbf{z}} = \alpha + I\tilde{\mathbf{z}} + \epsilon$ **where** $\tilde{\mathbf{z}} = \mathbf{z} - \bar{z}$ **and W is a row-standardised weight matrix**
     - ***Recall that*** $\mathrm{W}\tilde{\mathbf{z}}$ ***is a vector***
  4. **Bootstrap this procedure to estimate standard error (or use software)**

2. Spatial autocorrelation

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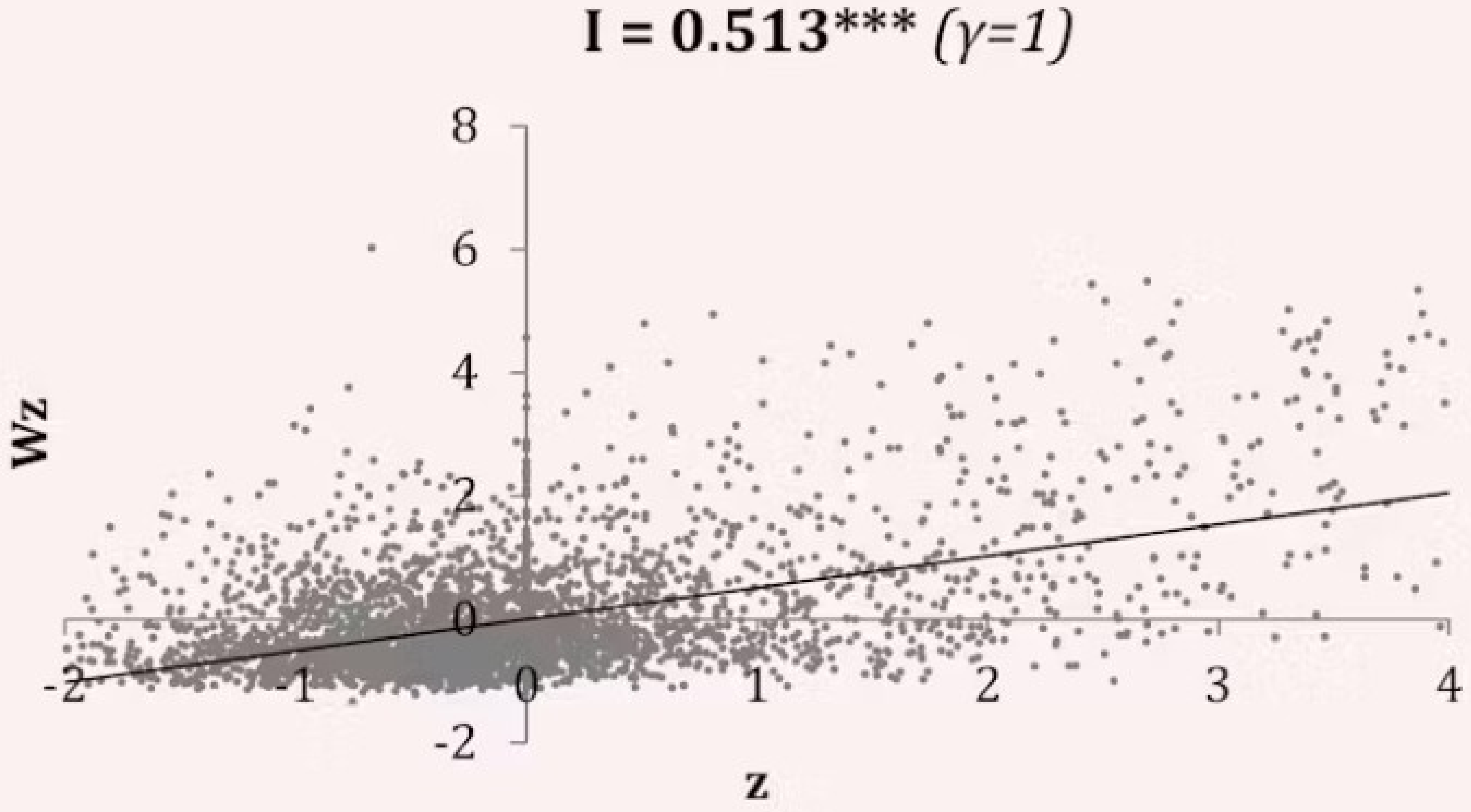
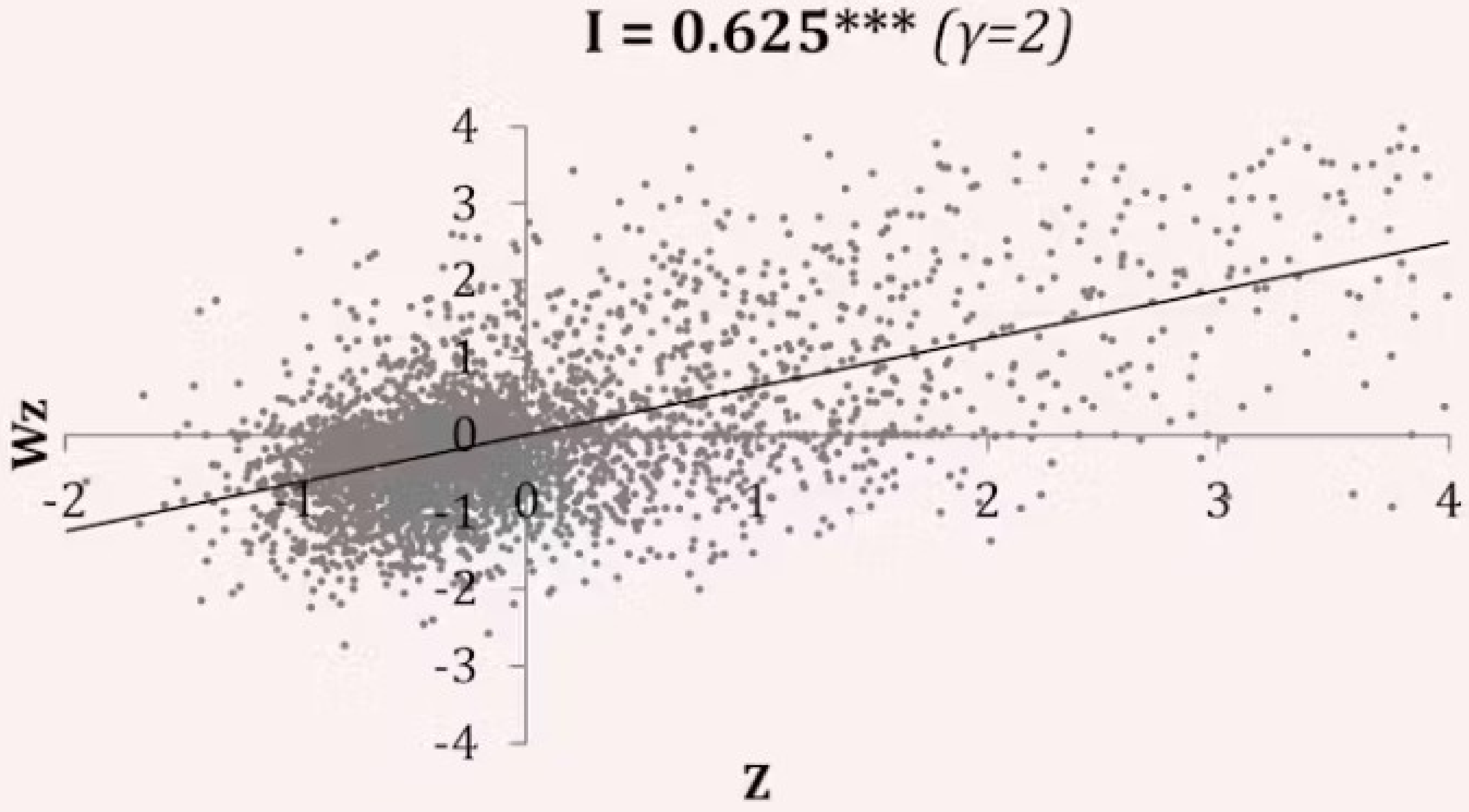
- **Calculate Moran's I**
  - **Using inverse distance function** $w_{ij} = \frac{1}{d_{ij}^{\gamma}}$





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- **Spatial correlation in deprivation**
  - **Local phenomenon?**
  - **You do not know *why* scores are autocorrelated...**
  - **No causal relationships!**

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2. Spatial autocorrelation

- It is important to make a distinction between *global* and *local* spatial autocorrelation
  - See Anselin (2003) for a discussion
- Global spatial autocorrelation
  - Local shock affects the whole system
- Local spatial autocorrelation
  - Local shock only affects the 'neighbours'

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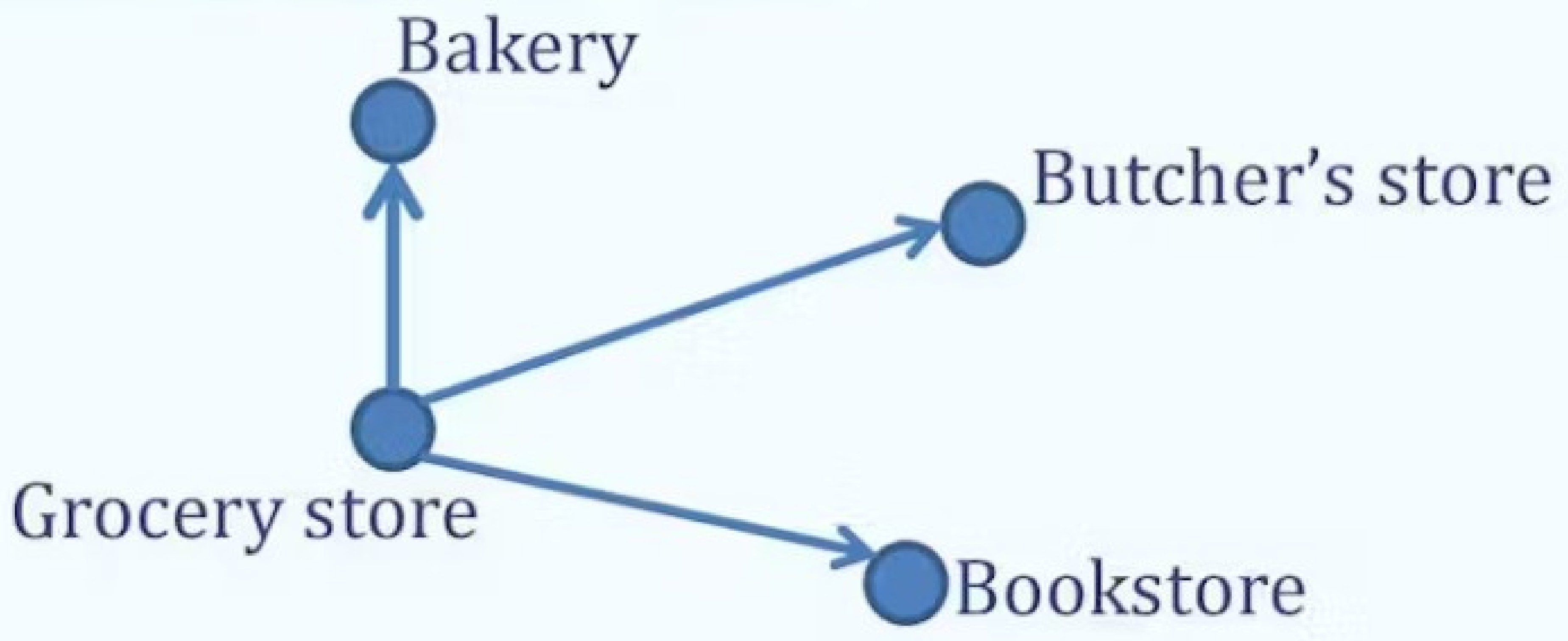
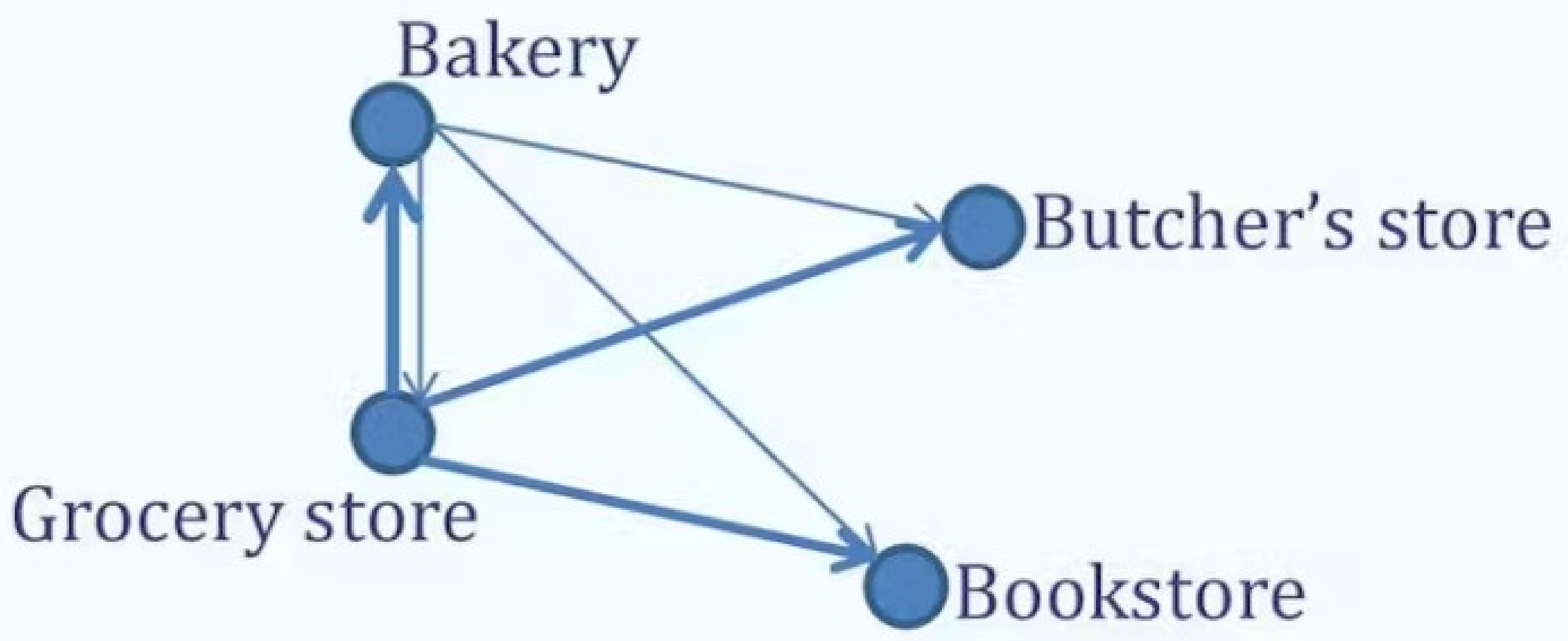
- **Example: Consider an income increase for grocery store owner**
- **Local autocorrelation:**



- **Global autocorrelation:**



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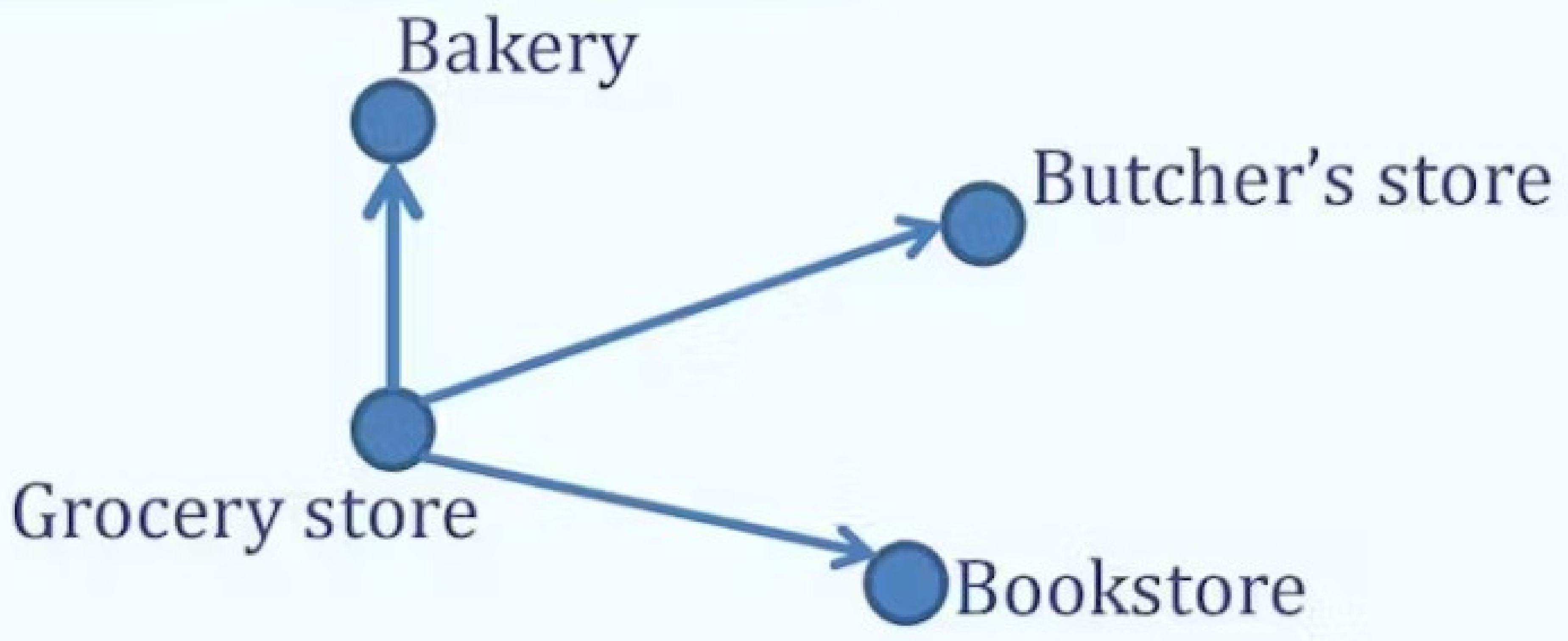
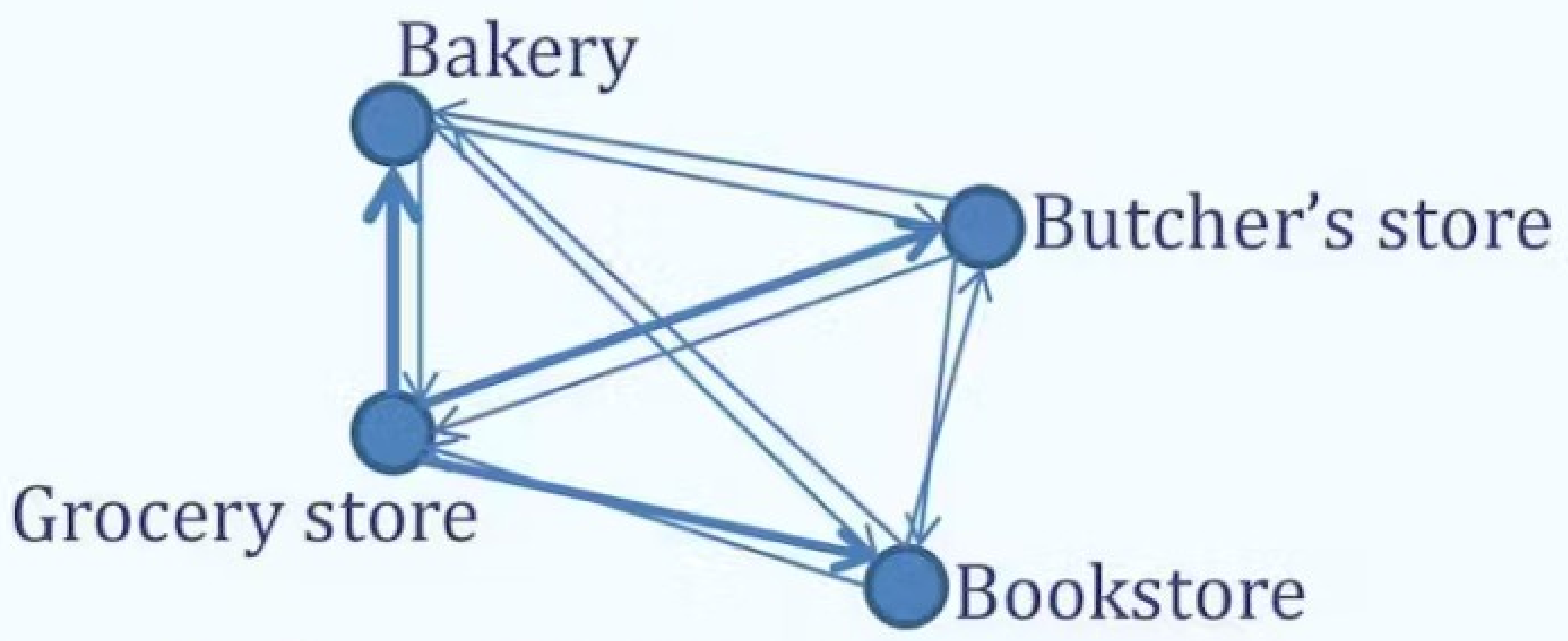
- **Example: Consider an income increase for grocery store owner**
- **Local autocorrelation:**



- **Global autocorrelation:**



… spatial multiplier process

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2. Spatial autocorrelation

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- Let's define $z = \lambda W z + \mu$
  - Reduced-form of z is $z = [I - \lambda W]^{-1} \mu$
  - With $\lambda < 1$

- A Leontief expansion yields:
  - $[I - \lambda W]^{-1} = I + \lambda W + \lambda^2 W^2 + \lambda^3 W^3 + \cdots$

- $W^2 \rightarrow$ There is an impact of neighbours of neighbours (as defined in W) although it is smaller (λ^2)
  - Global autocorrelation
  - *Spatial multiplier* process
  - In practice: covariance may approach zero after a relatively small number of powers

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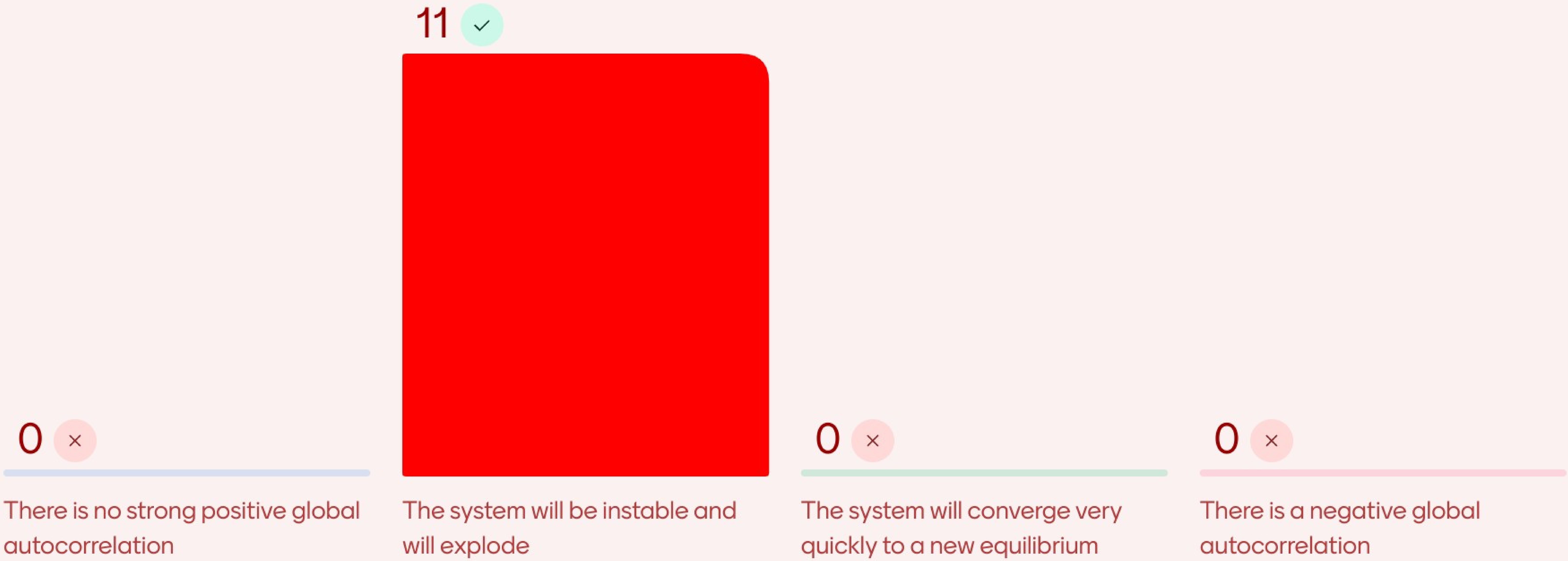
What happens when $\lambda > 1$ in $\mathbf{z} = \lambda \mathbf{W} \mathbf{z} + \mu$?



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- Let's define $z = \lambda W \mu + \mu$
  - This is already a reduced-form of z
- No impact of behaviour beyond 'bands' of neighbours
  - Dependent on definition of W
  - ...Local autocorrelation
- Covariance is zero beyond these bands

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- Local or global autocorrelation?
  - Dependent on application
  - Theory...

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- **Taxonomy:**

$$y = \rho W y + X\beta + W X \gamma + \epsilon \quad (1)$$

with

$$\epsilon = \lambda W \epsilon + \mu \quad (2)$$

W is a row-standardised weight matrix

$\rho, \gamma, \beta, \lambda$ are parameters to be estimated

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3. Spatial regressions

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- **Taxonomy:**

$$\boxed{y} = \rho W y + \boxed{X\beta} + W X \gamma + \epsilon \qquad (1)$$

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$$y = \rho W y + X\beta + \boxed{WX\gamma} + \epsilon \quad (1)$$

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$\rho, \gamma, \beta, \lambda$ **are parameters to be estimated**

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- **Taxonomy:**

$$y = \rho W y + X\beta + WX\gamma + \epsilon \quad (1)$$

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W is a row-standardised weight matrix

$\rho, \gamma, \beta, \lambda$ **are parameters to be estimated**

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- **Spatial lag model**
  - $\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X} \boldsymbol{\beta} + \boldsymbol{\mu}$ (3)
  - $\rho \neq 0, \boldsymbol{\gamma} = 0, \lambda = 0$
  - **Spatial dependence in dependent variables**

- **Note similarity with time-series models**
  - **AR Model**
  - $\boldsymbol{y}_t = \rho \boldsymbol{y}_{t-1} + \boldsymbol{X}_t \boldsymbol{\beta} + \boldsymbol{\mu}_t$ (4)

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- **Spatial lag model**
  - $\boldsymbol{y} = \rho \boldsymbol{W} \boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu}$ (3)

- **The outcome variable influences everyone (indirectly)**
  - **Global autocorrelation**

- **We may write**
  $(\mathbf{I} - \rho \boldsymbol{W})\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$
  $\boldsymbol{y} = (\boldsymbol{I} - \rho \boldsymbol{W})^{-1}(\boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\mu})$ **with**
  $(\boldsymbol{I} - \rho \mathbf{W})^{-1} = \boldsymbol{I} + \rho \boldsymbol{W} + \rho^2 \boldsymbol{W}^2 + \rho^3 \boldsymbol{W}^3 + \cdots$

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Can the spatial lag model $\mathbf{y} = \rho \mathbf{W} \mathbf{y} + X\beta + \mu$ be estimated by OLS?

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- **Spatial lag model**
  - $y = \rho W y + X\beta + \mu$ (3)

- **We cannot estimate this by OLS because of reverse causality**

- **Recall AR-model:**

  $y_t = \rho y_{t-1} + X\beta + \mu_t$ (4)
  - **We can estimate this in principle by OLS because y_{t-1} is not influenced by y_t**

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- **Spatial lag model**

- **Estimate with OLS?**
  - **Let's simplify** (3) **to**
    $$\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\mu} \qquad (3')$$

- **The estimator for** ρ **yields:**
  $$\hat{\rho}_{OLS} = \frac{(\boldsymbol{W}\boldsymbol{y})'\boldsymbol{y}}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

→ **Show that** $\hat{\rho}_{OLS}$ **is biased when** $\mathrm{cov}(\boldsymbol{y}, \boldsymbol{\mu}) \neq 0$

Consider estimating $\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mu$ by OLS. Show that ρ_{OLS} is biased when $\mathbf{cov}(\mathbf{y}, \mu) \neq 0$.

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I am ready!

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I am stuck...

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- **Spatial lag model**

- **Estimate with OLS?**
  - **Let's simplify (3) to**

$$\boldsymbol{y} = \rho \boldsymbol{W}\boldsymbol{y} + \boldsymbol{\mu} \qquad (3')$$

- **The estimator for ρ yields:**

$$\hat{\rho}_{OLS} = \frac{(\boldsymbol{W}\boldsymbol{y})'\boldsymbol{y}}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

- **If we plug-in (3') we get:**

$$\hat{\rho}_{OLS} = \frac{(\boldsymbol{W}\boldsymbol{y})'(\rho\boldsymbol{W}\boldsymbol{y}+\boldsymbol{\mu})}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

$$\hat{\rho}_{OLS} = \rho + \frac{(\boldsymbol{W}\boldsymbol{y})'\boldsymbol{\mu}}{(\boldsymbol{W}\boldsymbol{y})'(\boldsymbol{W}\boldsymbol{y})}$$

- **Hence, when $\mathrm{cov}(\boldsymbol{y}, \boldsymbol{\mu}) \neq 0$, $\hat{\rho}_{OLS}$ is biased**

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- Spatial lag model

- Use maximum likelihood (ML) estimator
  - Selects the set of values of the model parameters that maximizes the likelihood function

- Instrumental variables (IV)
  - Instruments for y may be WX and $W^2\mathrm{X}^2$
  - Less efficient than ML, but feasible for 'large' datasets
  - *e.g.* Kelejian and Prucha (1998)

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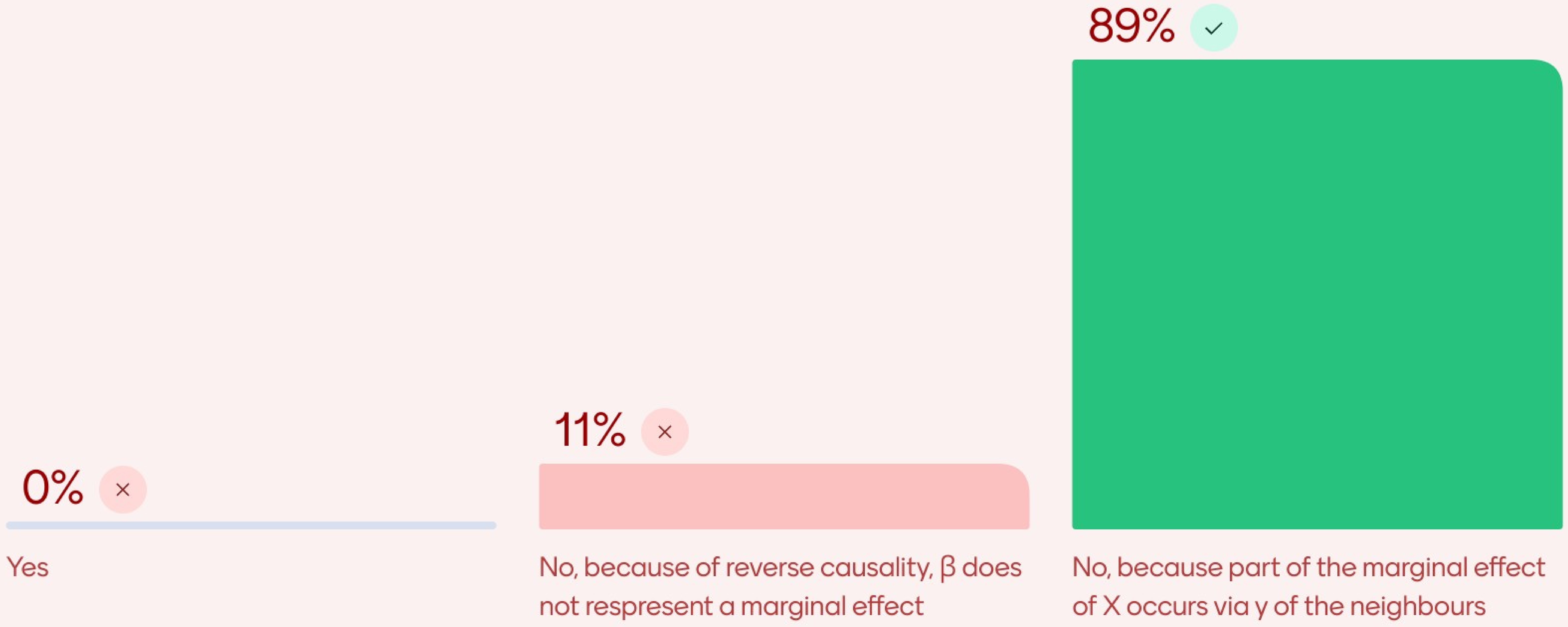
Assume you use Maximum Likelihood. Does β represent a marginal effect in a spatial lag model $\mathbf{y} = \rho \mathbf{W} \mathbf{y} + X\beta + \mu$?



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3. Spatial regressions

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- **Spatial cross-regressive model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\gamma W X} + \boldsymbol{\mu}$ (5)
  - $\rho = 0, \boldsymbol{\gamma} \neq 0, \lambda = 0$

Can the spatial cross-regressive model $\mathbf{y} = X\beta + \gamma\mathbf{WX} + \mu$ be estimated by OLS?

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- **Spatial cross-regressive model**
  - $$y = X\beta + \gamma WX + \mu \qquad (5)$$

- **Include (transformations) of exogenous variables in the regression**
  - **OLS is fine!**

- **Autocorrelation is local**

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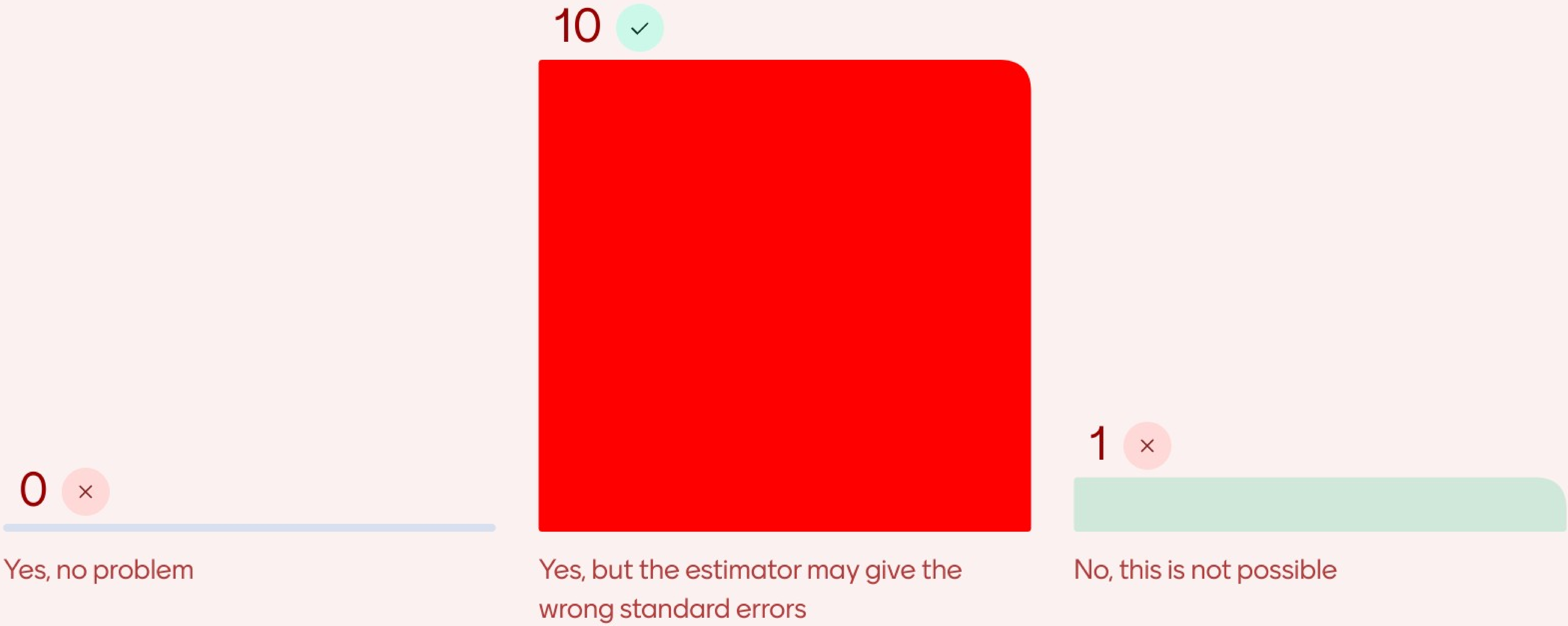
- **Spatial error model**
  - $\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \text{ with } \boldsymbol{\epsilon} = \lambda \mathbf{W}\boldsymbol{\epsilon} + \boldsymbol{\mu}$ (6)
  - $\rho = 0, \boldsymbol{\gamma} = 0, \lambda \neq 0$

Can the spatial error model $\mathbf{y} = X\beta + \lambda \mathbf{W}\epsilon + \mu$ be estimated by OLS?



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- Spatial error model
  - $y = X\beta + \epsilon, \text{ with } \epsilon = \lambda W\epsilon + \mu$ (6)

- Omitted spatially correlated variables
  - e.g. Ad-hoc defined boundaries
  - Uncorrelated to X!

- Consistent estimation of parameters β
- But: inefficient
  - ϵ are not i.i.d.
  - Standard errors are higher in OLS
  - β may be different in 'small' samples

3. Spatial regressions

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- **How to apply these models in practice?**

- SPAUTOREG **in STATA**
- SPATREG **in STATA**
- **GeoDa (free software, also for large datasets)**
- PACE'S SPATIAL STATISTICS TOOLBOX **in MATLAB**

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Today:

- Test spatial autocorrelation using Moran's *I*
- Local vs. global spatial autocorrelation
- Incorporate space in regression framework
- Spatial regressions
  - Spatial lag model
  - Spatial cross-regressive model
  - Spatial error model

Spatial econometrics (2)

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